MODAL ANALYSIS FOR VOLTAGE STABILITY: Application at Base Case and Point of Collapse

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1. INTRODUCTION

Voltage stability has become an important issue to many power systems around the world, the large Brazilian interconnected system being no exception. There is a great interest in the development and application of computational tools and methodologies to voltage stability problems detected in power system planning and operation studies [1-31]. Various methodologies have been proposed in the last decade, together with system models to properly simulate the voltage stability phenomena.

Considering the lenght of this paper, it is appropriate to list its contributions at this initial stage:

- A multivariable control framework is provided to the modal analysis methodology established in [1].
- Specialized use of modal sensitivities for better problem identification and proposed solutions.
- First reported use of the concept of transfer function zeros to the voltage stability area.
- A method to determine suitable locations for Static VAr Compensators (SVC) and their required droops (gains in pu/pu) to help preventing voltage collapse.
- Voltage stability (modal) analysis of a large system containing over two hundred induction motor loads, each represented by a first-order motor model.
- Determination of required minimum size for a system model to properly simulate the behaviour of a given voltage collapse mode.
- Effective use of modal analysis at (or near to) the point of collapse [9].
- Report on the use of Point of Collapse and Continuation Power Flow programs.
- Describes the stage of development of the Brazilian voltage stability tools and the future R&D work to be jointly carried out by CEPEL, ELETROBRÁS and other Brazilian utilities.

The results described in the paper are for a large practical model of the South-Southeast Brazilian system. The operating point considered is intended to duplicate real system conditions of March 1993, when a blackout, due to voltage collapse, took place in the Rio de Janeiro area.

Keywords: *Voltage Stability - Modal Analysis - Large Scale Systems - Continuation Methods - Power Flow - Point of Collapse Methods - Transfer Function Residues - Eigenvalue Sensitivities - Maximum Loadability - Static VAr Compensators - Induction Motor Loads.*

2. VOLTAGE STABILITY METHODS

2.1. Modal Analysis of Small Signal Voltage Stability

Modal analysis can be used to determine which areas are most vulnerable to voltage stability problems, to select the best sites for installing new reactive compensation equipment, and to determine the most effective actions such as generation redispatch and load shedding to alleviate voltage conditions.

The user must always recognize that modal analysis results are rigorously valid for incremental changes only. The power of this methodology, therefore, lies in its ability to provide information on system trends rather than estimate the actual numerical values of system variables following changes.

The work reported in this paper gives a multivariable control framework to the methodology established in [1]. The methodology of [1] performs modal analysis on the state matrix $\left(J_R^Q\right)$. One can alternatively obtain the same information from the eigensolution of $\left(-J_R^Q\right)^{-1}$, which can be expressed as:

$$sI+\left(J_R^Q\right)^{-1}=0 \qquad (1)$$

where **I** is the identity matrix and $\left(J_R^Q\right)$ is the reduced Jacobian matrix defined in [1], and also known as the Q-V sensitivity matrix.

Good insight is obtained by recognizing that equation (1) describes the dynamics of the system model represented by the block diagram of Figure 1:

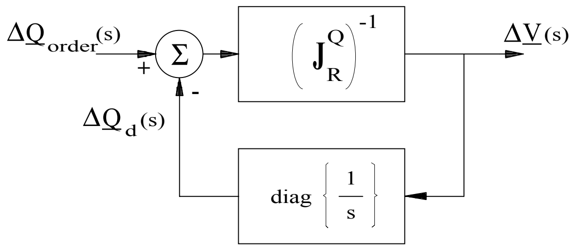


Figure 1. Simple Dynamic Load Model for Voltage Stability Analysis of Large Systems

The elements of the diagonal matrix in the feedback loop of Figure 1 are pure integrators. The symbol $\Delta \underline{V}(s)$ denotes the vector of bus voltage magnitudes and $\Delta Q_{order}(s)$ is the input vector of specified changes in reactive power for the various loads in the system. State vector $\Delta Q_d(s)$ contains the incremental changes in reactive demand of the load buses, whose active powers are independent of voltage deviations (constant MW loads).

This model has some analogy to the linearized form of the classical multimachine electromechanical stability problem, described by equations:

$$s^2 \mathrm{I} + diag\left\{\frac{\omega_o}{2\mathrm{H}_i}\right\} \mathrm{K}_s = 0 \qquad (2)$$

where $\mathbf{K_s}$ is the matrix of synchronizing torque coefficients [32].

The eigenvalues of the dynamic model of Figure 1 are those of matrix $\left(-\mathrm{J}_R^Q\right)^{-1}$. A system eigenvalue therefore goes to infinity at the voltage stability limit where
det $\left(\mathrm{J}_R^Q\right) = 0$.

The dynamic load model of Figure 1 is very useful despite not being meant to reflect actual load dynamics. The authors of [28] made a comment on this model in their closure, which is here quoted: "The development of this simple dynamic load model has made it possible to relate the eigenvalues of $\left(\mathrm{J}_R^Q\right)$ to the poles of the closed-loop transfer function of the dynamic system".

2.1.1. Identifying Critical Buses, Branches and Generators Via Residues and Eigenvalue Sensitivities (Fast Ranking)

Small signal stability assessment of large power systems can be efficiently computed when using an augmented form of the system state-space description [48,49,50]. For the sake of brevity let us here simply define **A** as the system state matrix and $\underline{\mathbf{b}}$ and $\underline{\mathbf{c}}^t$ as the vectors for a given input/output pair.

Once the eigenvalues of matrix $\left(\mathrm{J}_R^Q\right)^{-1}$ are calculated, valuable information on corrective measures can be obtained through transfer function residues [32], participation factors [1,39] and eigenvalue sensitivities. Residues can be used to determine best locations for dynamic voltage devices among other uses.

A scalar function **F**(s) can be expressed as [32]:

$$\mathrm{F}(s) = \frac{\mathrm{N}(s)}{\mathrm{D}(s)} = \underline{c}^t \cdot (s\mathrm{I} - \mathrm{A})^{-1} \cdot \underline{b} = \sum_i^n \frac{\mathrm{R}_i}{s - \lambda_i} \qquad (3)$$

where integer *n* is the dimension of **A** and R_i is the residue of **F**(s) associated with the i-th eigenvalue of **A**. The residue can be expressed as

$$\mathrm{R}_i = \tilde{b}_i \cdot \tilde{c}_i \qquad (4)$$

where $\tilde{b}_i = \boldsymbol{\xi}_i^t \cdot \underline{\mathbf{b}}$ is the mode controllability factor and $\tilde{c}_i = \underline{\mathbf{c}}^t \cdot \boldsymbol{\eta}_i$ is the mode observability factor. Note that $\boldsymbol{\xi}_i$ and $\boldsymbol{\eta}_i$ are, respectively, the left and right eigenvectors associated with λ_i.

Once the desired right and left eigenvectors are calculated, the residues for a large number of transfer functions can be efficiently obtained through the use of the corresponding augmented input/output vectors [32].

A general formula for eigenvalue sensitivity coefficients applied to the generalized eigenvalue problem was described in [37].

Useful information can be obtained through eigenvalue sensitivity analysis when α is chosen to be a network impedance, a dynamic component parameter, etc.

The differences between eigenvalue sensitivities $\frac{\partial \lambda}{\partial \alpha}$, residues and participation factors are described in [3,5]. Participation factors have been the ones mostly used in the voltage stability area [1,9,24]. The authors of this paper support a more specialized use of modal sensitivities, such as that proposed in Table 1. Various functions are listed in Table 1 together with their recommended uses when attempting to stabilize a critical voltage mode. The computation of these sensitivities can be made very fast through clever sparsity programming. Scaling problems should be recognized and properly dealt with when producing ranking lists out of modal sensitivities.

2.1.2. Transfer Function Zeros

The zeros of the transfer function of equation (3) can be obtained by solving the generalized eigenvalue problem [42]:

$$\begin{pmatrix} \mathbf{A} & \underline{b} \\ \underline{c}^t & 0 \end{pmatrix} \cdot \begin{pmatrix} \underline{x} \\ u \end{pmatrix} = \lambda \cdot \begin{pmatrix} \mathrm{I} & \underline{0} \\ \underline{0}^t & 0 \end{pmatrix} \cdot \begin{pmatrix} \underline{x} \\ u \end{pmatrix} \qquad (5)$$

where **I** is the identity matrix, $\underline{\mathbf{0}}^t$ is a row vector with all elements equal to zero and u is the input variable. A standard QZ routine [51] can be used to obtain the complete set of transfer function zeros, as long as the system is of moderate size.

Subspace iterations methods can be used to efficiently calculate several zeros at a time of large power system dynamic models. These methods must be applied to the augmented generalized eigenvalue problem, which is sparse, and is described in [52]. An application of the concept of transfer function zeros to the voltage stability problem is described in section 5.3.2 of this paper.

Residues

$\frac{\Delta V}{\Delta B_{sh}}(\lambda)$	Find best buses to place Static VAr Compensators
$\frac{\Delta V}{\Delta Q_{sh}}(\lambda)$	Find best buses to place synchronous condensers, generators and advanced Static VAr Compensators
$\frac{\Delta V}{\Delta B_{ij}}(\lambda)$	Find best lines to place Thyristor Controlled Series Compensation
$\frac{\Delta V}{\Delta \phi_{sh}}(\lambda)$	Find best lines to place static phase shifters

Participation Factors of ΔV

Define the component buses of reduced size system model to properly simulate a given voltage mode

Eigenvalue Sensitivities

$\frac{\partial \lambda}{\partial B_{sh}}$	Find best buses to connect fixed shunt compensation
$\frac{\partial \lambda}{\partial Bij}$	Find best lines to install fixed series capacitors
$\frac{\partial \lambda}{\partial tap}$	Find OLTC transformers where tap changes are most effective
$\frac{\partial \lambda}{\partial V}$	Find generators where voltage level changes are most effective
$\frac{\partial \lambda}{\partial Pg}$	Find generators where redispatch is most effective
$\frac{\partial \lambda}{\partial P_{load}}$	Find most critical active loads
$\frac{\partial \lambda}{\partial Q_{load}}$	Find most critical reactive loads

Table 1*: Modal Sensitivity Functions for Voltage Stability Analysis and Their Reccomended Uses.*

2.1.3. How to Stop Missing Unstable Eigenvalues in Partial Eigensolutions

In large scale power flow models, only partial eigensolutions of $\left(-J_R^Q\right)$ are possible. The computer program described in [1] computes a group of eigenvalues which have the smallest moduli, and therefore are of interest. However, sometimes, it might exist (in voltage unstable cases) positive eigenvalues (unstable) with not so small a modulus and, therefore, this approach may fail to detect these instabilities in a moderate number of computer runs.

A practical solution to this problem is to operate on matrix $\left(-J_R^Q\right)^{-1}$. A zero eigenvalue of $\left(-J_R^Q\right)$ is therefore mapped into infinity, while negative small values become large negative and small positive eigenvalues become large positive ones. The strategy is to perform partial eigensolutions on the spectral transformations $\left(-J_R^Q-\alpha I\right)^{-1}$ and $\left(-J_R^Q+\alpha I\right)^{-1}$, where **I** is the identity matrix and the real shift α is usually given a value around 5 or larger for near collapse situations. This solution leads to an apparently larger number of computer runs, but the chances of missing unstable eigenvalues are practically eliminated.

2.1.4. Recommended Use for Modal Analysis

Some voltage stability experts [9,24], advocate the use of modal analysis mainly for highly stressed system conditions. The authors' experience confirms that modal analysis is most effectively used for system reinforcement studies when made at the point of collapse (maximum loadability point) or near it, as will be shown in a later section of this paper. Modal analysis can not be used to predict system loadability margins. Such function must be left to Continuation and Point of Collapse Power Flow programs. Voltage stability margins as functions of the nature of dynamic loads can however be assessed by modal analysis based even on a single power flow condition (see Table 7 of this paper).

2.2. Continuation and Point of Collapse Power Flows

These two methodologies, both of which enhance the power flow analysis area, are detailed described in [17,18,38,31]. The Continuation method makes the Newton Power Flow very robust, allowing convergence in all cases that have a solution in the real domain. The Point of Collapse method obtains, on a single run, the maximum system loadability point for a specified pattern of load changes. The latter method is specially useful when used in conjunction with a modal analysis tool (see section 2.1.4).

A recently developed function to Point of Collapse Power Flows is the computation of the locally closest bifurcations in power systems. The distance in load parameter space to this closest bifurcation is a good index of voltage collapse and a minimum load power margin [14].

3. SYSTEM MODELING FOR VOLTAGE STABILITY ANALYSIS

3.1. Induction Motor Model

The load model of Figure 1 is only meant to be used in connection with steady state power flow problems: maximum loadability; maximum transferability between system areas; transmission and generation reinforcements; allocation of VAr compensation; assessment of voltage critical areas in large system models; etc.

In order to have a more realistic simulation of the voltage dynamic behavior, more detailed models for load and voltage control equipment are needed [2,4,5,7,8,25,26]. Aperiodic voltage stability was shown to be correctly assessed from first-order load models [16]. Slip from synchronous speed is the state variable adopted in the first-order induction motor model of this paper. The block diagram for a linearized system comprised of a generic number of first-order induction motor loads is shown in Figure 2.

The symbol H_i in Figure 2 denotes the "i-th" motor inertia constant, while $\Delta\underline{P}_{mec}$ and $\Delta\underline{P}_{el}$ are vectors containing the motor mechanical and electrical powers in synchronous watts. Matrix $[\partial\underline{P}_{el}/\partial\underline{s}_{lip}]$ describes the incremental relationships, at a given operating point, between electrical power requirements and rotor slip for all motors in the system.

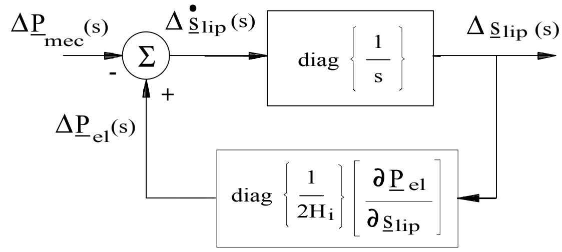


__Figure 2.__ Block Diagram Model for Linearized System Comprised of a Generic Number of Induction Motor Loads

The voltage stability limit occurs at the condition of maximum power transfer to the induction motor. Therefore, the voltage stability limit for a motor (or group of motors) occurs exactly at motor (aggregate motor model) pull-out torque conditions considering the entire transmission network. The latter fact, was initially described in [44] under section "Stability of Loads".

Some of this paper's results consider induction motor loads and the action of static VAr compensators (SVC). Table 5 of this paper presents the critical eigenvalues associated with the voltage collapse phenomena in the RIO AREA as a function of the SVC location. Voltage stability results for a large system having 239 induction motors are given in Table 7 of this paper.

3.2. Making a Conventional Small Signal Electromechanical Stability Program Suitable to the Analysis of Voltage Stability Problems

Eigenanalysis tools for voltage stability have been developed around the power flow Jacobian matrix, with voltages expressed in polar coordinates, calculated at specified operating points. This is the easiest route for the code development of a first generation program, with the same level of modeling as a conventional power flow. Load, generator and voltage support equipment models of higher complexity are, however, needed in a more accurate simulation of voltage stability phenomena, and the conventional power flow Jacobian matrix is no longer the most convenient network model to use. A more familiar network model to power system dynamics specialists is the nodal admittance matrix, used in all production type programs for electromechanical stability analysis.

The major advantage of using a comprehensive small-signal electromechanical stability program for voltage stability software development is the ready availability of models for the elements which drive the system voltage response (loads, induction motors, HVDC links, etc.) and for those which provide the voltage support (Generators and their excitation control, Synchronous Condensers, Static VAr Compensators, FACTS devices, etc.).

The connection of system devices (Generators, Loads, etc.) to the electrical network, in small-signal electromechanical stability programs, is expressed through current injections to the associated buses in the nodal admittance matrix.:

$$\Delta\mathbf{I} = \mathbf{F}.\Delta\mathbf{x} + \mathbf{Y}.\Delta\mathbf{V} \tag{6}$$

where $\Delta\mathbf{I}$ is comprised of individual injections ΔI^k_{real} and ΔI^k_{imag} , $k=1,...,n_{devices}$, $\Delta\underline{\mathbf{x}}$ is the system state vector, and

$$\mathbf{F} = \frac{\partial \underline{\mathbf{I}}}{\partial \underline{\mathbf{x}}} = \text{block diag}\left\{\mathbf{F}_i, i = 1, \ldots, n_{devices}\right\} \tag{7}$$

Small-signal voltage stability analysis usually deals with models based on the incremental active and reactive power changes to the system buses. The modeling of the system of Figure 1, on a small-signal electromechanical stability program, can be done through the use of the incremental relationships below:

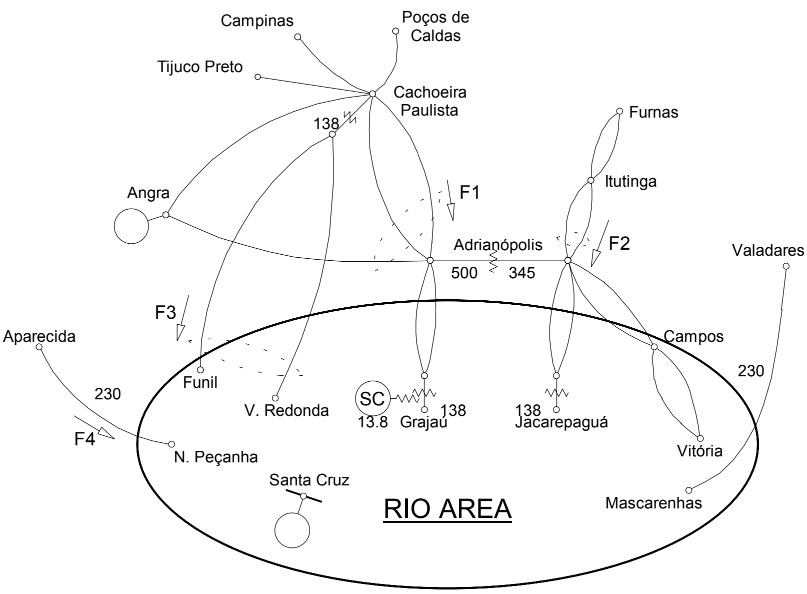
$$\begin{pmatrix} \Delta\mathbf{I}^k_{imag} \\ \Delta\mathbf{I}^k_{real} \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbf{I}^k_{imag}}{\partial P^k_d} & \frac{\partial \mathbf{I}^k_{imag}}{\partial Q^k_d} \\ \frac{\partial \mathbf{I}^k_{real}}{\partial P^k_d} & \frac{\partial \mathbf{I}^k_{real}}{\partial Q^k_d} \end{pmatrix} \begin{pmatrix} \Delta P^k_d \\ \Delta Q^k_d \end{pmatrix} \tag{8}$$

where ΔP^k_d and ΔQ^k_d are the incremental changes in the active and reactive load demands at the *k*-th bus.

4. LARGE SYSTEM MODEL UTILIZED IN THE ANALYSIS OF A REAL INCIDENT OF VOLTAGE COLLAPSE

The main results presented in this paper are for a 1663-bus model of the South-Southeast interconnected, predominantly hydro-powered, Brazilian system. The base case operating point attempts to duplicate real system conditions of March, 1993, when a blackout, due to voltage collapse problems, took place in the Rio de Janeiro area. The voltage collapse was induced by the permanent loss of two 500kV lines (from C. Paulista to Adrianópolis substations). These two major lines remained disconnected for several days and the power utilities of the RIO AREA, which encompasses Rio de Janeiro and Espírito Santo States, held an intensive campaign for voluntary reduction in power consumption. This campaign was quite successful and the power flow into the RIO AREA fell to presumably safe values.

A schematic diagram of the area in focus is presented in Figure 3. The total power flowing into the RIO AREA, denoted by FRJ, is given by the summation of flows *F1*, *F2, F3* and *F4* which are indicated in Figure 3. The total active load in the area is about 5000MW, while the local hydro generation is about 900MW. There is one thermal power station (Santa Cruz) in the Rio area with a generating capacity of 600MW (2x220 plus 2x84), which can not be operated in a synchronous condenser mode. It is desirable, due to the high fuel costs, to operate the thermal plant at a minimum level. Dynamic security studies have established a minimum thermal generation of 45MW under heavy load conditions.



Note: Numbers shown denote transmission voltages in kV

Figure 3. *Major Transmission Delivering Power to the RIO AREA*

5. RESULTS

5.1. Conventional Power Flow Results

The base case system configuration described in section 4 together with some credible contingencies (line outages and generation outages) were considered. The conventional power flow results are displayed in Figure 4, in the form of P-V curves, where V denotes the voltage magnitude of a critical system bus (Jacarepaguá) and P is the total active power flowing into the RIO AREA (FRJ). All these curves were obtained by keeping the Grajaú synchronous condenser generation below its MVAr limit (400 MVAr). This synchronous condenser group (2 x 200 MVAr) is a key element in ensuring safe voltage stability margins to the RIO AREA.

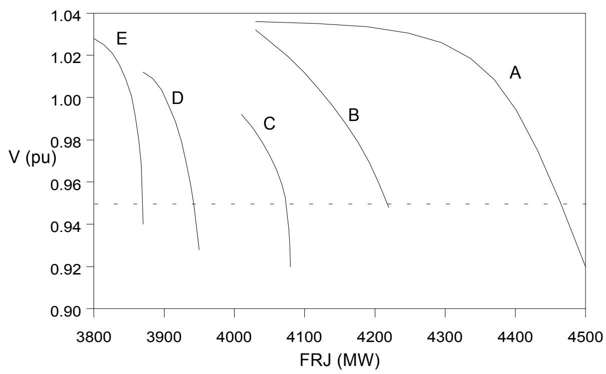
The maximum power import values, obtained from the analysis of Figure 4, are shown in Table 2 for the various conditions simulated.

Table 2: *Approximate Maximum Values of Power Import by RIO AREA (FRJ)*

Case	FRJ (MW)	System Condition	
		Angra Nuclear Station	500kV Line Outage
A	4450	ON	NONE
B	4230	OFF	NONE
C	4070	ON	Angra-Adrianópolis
D	3950	OFF	Angra-Adrianópolis
E	3860	OFF	C. Paulista-Adrianópolis
F	4200	ON	C. Paulista-Adrianópolis

Note: 1) Values determined by assuming 0.950 pu as the minimum allowed voltage level at critical bus Jacarepaguá 345kV (pilot node)

2) The Santa Cruz thermal station generates 45MW in all cases



System Conditions A, B, C, D and E are described in Table 2

Figure 4: *Voltage Level at Pilot Node (Jacarepaguá bus) as a Function of MW Power Imported by RIO AREA*

5.2. Modal Analysis at a Given Operating Point

The 1663-bus system has a state matrix of order 1554, and therefore the same number of eigenvalues. The QR eigensolution of such a large matrix is not practical and one must use sparsity programmed partial eigensolution routines. As mentioned in section 2.1.3, the state matrix used in the authors' program is $\left(-J_R^Q\right)^{-1}$ and, therefore, the eigenvalues of interest are those of largest moduli and negative sign, together with all those which are positive (unstable).

Some computer runs utilizing a lop-sided simultaneous iteration algorithm [1,33] with different shifts produced, in a few minutes of CPU on a PC 486, all of the most critical system eigenvalues/eigenvectors. Three of the most critical ones are depicted in Table 3. From eigenvector information (mode-shape of bus voltage magnitudes), the critical eigenvalues are seen to be associated with voltage modes in clearly defined areas of the large interconnected system. Two of these areas are known to power system operations engineers as problem areas regarding power flow convergence. Actually, this base case power flow problem only converges when modeling a good part of the loads in these two areas as voltage dependent (constant I and constant Z load models). Note that the power flow Jacobian is practically singular (λ=-19.784 and λ=-41.896)., since the Corumbá and Serra da Mesa loads were modeled as constant P in the voltage stability analysis program.

It is worth reminding that this paper's results are obtained from the inverse of the Q-V sensitivity matrix of [1], and therefore its eigenvalues are the inverse of those obtained with the methodology of [1].

Modal analysis helps developing voltage control strategies to better converge other power flow cases derived from the base case analyzed. The results of Table 3 make one very much aware of the multiple and simultaneous quasi-voltage collapses contained in a base case power flow of a large interconnected system.

Critical Eigenvalues	Problem Area *	D** (km)
-19.784	Serra da Mesa (Celg/Furnas)	1000
-41.896	Corumbá (Enersul)	1500
-3.5686	RIO AREA	0

*System Area where Mode is Mostly Observable
**Approximate Distance of Problem Area from RIO AREA
Note: All system eigenvalues (total of 1554) are stable

Table 3: *Critical Eigenvalues of* $\left(-\mathrm{J}_R^Q\right)^{-1}$ *Matrix for Base Case Power Flow*

5.2.1. Determining a Model of Minimum Size for the Voltage Stability Analysis of a Given System Area

The Q-V sensitivity (state) matrix of the power flow Jacobian contains as many lines as the full set of PQ buses in the system. A critical mode, in voltage stability analysis of practical systems, is generally associated with a specific system area rather than being of a global nature (note that, in the electromechanical stability problem, the inter-area modes are of a global nature). This suggests the possibility of reducing the state matrix utilized in the analysis of phenomena associated with a given system area. The reduced Q-V matrix will then only contain the equations for the buses with significant participation factors in the critical modes of the area (internal system). One can perform elimination (Kron's reduction) on the equations for the external system buses, but this should not be done explicitly for the sake of preserving sparsity. The practical solution is to always work with the full size system Jacobian. If different shifts are specified to find all critical eigenvalues, routines for sparse partial refactorization should be applied to operate on the internal system factorization tree [40].

Table 3 showed the three most critical modes identified from the partial eigenanalysis of the 1554th-order state matrix of the 1663-bus system. Table 4 shows that these modes can be obtained and further analyzed from highly reduced Q-V state matrices. The quality of the approximate model is assessed from the bus voltage mode shape or participation factors information and not from the numerical value of the eigenvalue. The quality of the reduced model is, above all other factors, determined from its ability to present about the same maximum area loadability point as the full size model. One must ensure that there exists connectivity among all buses represented in the Q-V state matrix. Otherwise, the dynamic models in isolated portions of the network will interact through the algebraic part of the network, producing some spurious eigenvalues, which may bring difficulties to the interpretation of results.

Eigenvalue	Utilities Represented in State Matrix	No. of States	Quality of Approximate Model
RIO AREA Mode			
-3.5686	All Utilities	1554	-
-3.5626	Escelsa, Furnas, Light, Cerj, Cemig	448	Excellent
-3.5209	Escelsa, Furnas, Light, Cerj	331	Excellent
-2.8175	Escelsa, Furnas, Light	279	Good
-3.5970	Escelsa, Furnas, Cerj	216	Excellent
-2.8197	Escelsa, Furnas	164	Good
-1.8430	Escelsa	53	Fair
Serra da Mesa Mode			
-19.784	All Utilities	1554	-
-20.113	Celg, Ceb, Furnas	227	Excellent
-13.342	Celg, Furnas	169	Good
+8.1381	Celg, Ceb	116	Fair
+9.4093	Celg	58	Fair
Corumbá Mode			
-41.896	All Utilities	1554	-
-41.693	Enersul, Eletrosul, Cesp, Copel	369	Excellent
-37.879	Enersul, Eletrosul, Cesp	232	Excellent
+1029.7	Enersul, Eletrosul	125	Excellent
+16.71	Enersul	60	Good

Table 4: *Analysis of Critical Area Modes Using Reduced Size State Matrices*

A practical advantage of using reduced dynamic models is that full eigensolution becomes then possible through use of a standard QR library routine [41]. Another advantage is the reduction in the numbers of eigenvalues per cluster in large Q-V matrices, since the constituent eigenvalues of a single cluster may belong to numerous voltage areas, which can be entirely decoupled from that one under focus.

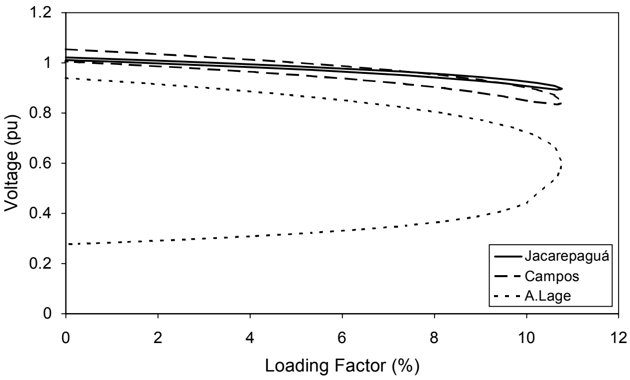


Figure 5: *System with no Q-limits*

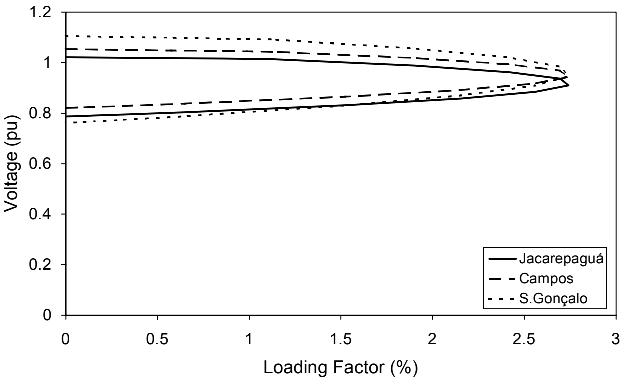


Figure 6: *Q-limits active only at Grajaú Synchronous Condenser (Grajaú limited at +400MVAr)*

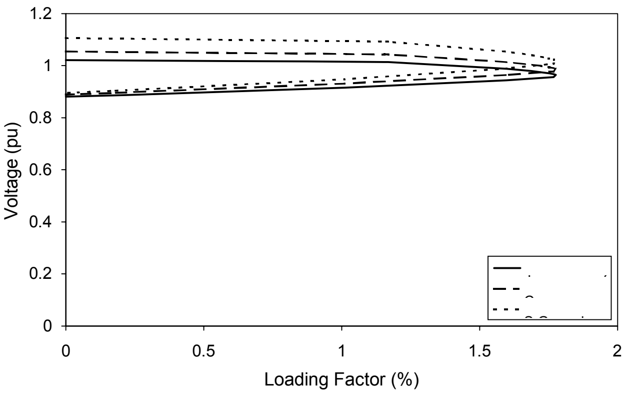


Figure 7: *Q-limits active at all PV Buses of the System*

5.3. P-V Curves Produced by a Continuation Power Flow

The Continuation Power Flow and Point of Collapse programs used in the work of this paper were developed at the University of Wisconsin by Dr. Fernando Alvarado and associates [18,14,38,31].

Figures 5, 6 and 7 show the P-V curves for the voltage magnitudes at two major 345kV substations in the RIO AREA (Jacarepaguá and Campos), with the thermal power station (Santa Cruz) disconnected. Voltage profiles are also included for two of the weakest buses in the RIO AREA: A.Lage (Escelsa) and S. Gonçalo (Cerj). The horizontal axis in these curves is the extra loading (in percentage of the base case MW-MVAr loads) imposed to the base case system load. Power interchange controls, LTC action and maximum VAr limits were not considered, to obtain the nose curves of Figure 5.

Note that the system maximum loadability point is about 111% (base case loading is 100%) in Figure 5, with Grajaú Synchronous Condenser generating 1300MVAr, well above its rated continuous maximum limit of 400MVAr. Figure 6 nose curves were generated by applying the 400MVAr limit to the Grajaú condenser, and the maximum loadability drastically dropped to 102.7%. Actually, a different mode of collapse occurred at 102.7% loading as a result of the loss of voltage control action at the Grajaú bus. Figure 7 curves were obtained by considering proper MVAr limits in all PV buses of the system, and the maximum loadability dropped further to 101.7% loading. These results indicate that the synchronous condenser at Grajaú bus is a key element for the voltage stability of the RIO AREA.

5.3.1. Determination of Suitable Locations for New Voltage Control Devices (Fast Ranking Lists)

The bus with the largest magnitude in the ranking list of residues $\frac{\Delta V^i(s)}{\Delta B^i_{sh}(s)}$, for $s = \lambda_{critical}$, is initially taken as the best one for installing an SVC in order to move $\lambda_{critical}$ (see Table 1).

The system test chosen to validate the methodology proposed in Table 1 was to assume, in the base case condition, the Grajaú synchronous condenser, here generating 340MVAr, to be at VAr limits and, therefore, considered as a PQ bus.

The system, as seen from the comparative analysis of Figures 5 and 6, has its voltage stability margins significantly reduced. The most critical RIO AREA eigenvalue jumps from -3.5626 to -11.764 when the Grajaú condenser bus is changed from PV to the PQ type. The ranking list of $\frac{\Delta V(s)}{\Delta B_{sh}(s)}$, shown in Table 5, is used to define the best locations for a dynamic voltage controller device to improve the most critical RIO AREA mode (-11.764). Note that the high-voltage bus (Grajaú 500 kV) of the synchronous condenser comes first in this ranking list among all other 500 kV buses. Table 5 also shows the location of the least stable system eigenvalue after installing, one-at-a-time, a purely algebraic model of an SVC (SVC gain is 30 pu/pu at all buses) at each one of the listed buses.

Note that, as expected, the SVC action improves the stability of the RIO AREA mode. This validates the methodology proposed for the location of voltage-controlling devices in the system based on modal sensitivities.

Bus	kV Level	Residues of ΔV/ΔBsh	RIO AREA Eigenvalue	Zeros of ΔV/ΔBsh
none	-	-	-11.764	
Grajaú	500	0.30580	-5.5843	-3.1072
Adrianópolis	500	0.28540	-5.7093	-3.0510
Angra	500	0.22820	-6.7907	-3.9419
C. Paulista	500	0.16100	-7.4144	-3.7988
Taubaté	500	0.10380	-8.7119	-5.5181
Tijuco Preto	500	0.07507	-9.3180	-6.0962
Campinas	500	0.05644	-9.9089	-7.3855
P. Caldas	500	0.05831	-9.8482	-7.2441
Araraquara	500	0.02640	-10.7713	-8.8341
Marimbondo	500	0.00417	-11.5683	-10.6369
Jacarepaguá	345	†	-5.1186	-2.9985
Campos	345	†	-6.8121	-4.6254

Table 5: *Location of Critical RIO AREA Eigenvalue After SVC Installation at a Given Bus*

5.3.2. Assessing the Maximum Impact to System Stability of a Voltage Control Device at a Given Bus

This assessment can be done through the calculation of the critical system zeros [42] of the transfer functions associated with the various voltage control devices. The analysis results presented in this section are concerned only with the zeros of the $\frac{\Delta V^i(s)}{\Delta B^i_{sh}(s)}$ transfer function. Table 5 displays the most critical zero of this transfer function for various buses of the system. These zeros represent the location in the complex plane, for one of the system poles when installing an SVC of infinite gain to a given system bus. If a zero is sitting on top of a given eigenvalue, it means that an SVC at that bus is incapable to control such voltage mode.

The critical zeros and system eigenvalues after SVC addition, all shown in Table 5, should be compared with λ = -11.764. This comparative analysis allows the determination of adequate gain values for the SVC's, as described below for the Grajaú 500kV bus. The eigenvalue is -11.764 and -5.4666 before and after the SVC addition, respectively. As the -5.4666 value is comparatively close to λ=-3.1072 which needs an SVC of infinite gain, one can conclude that K=30 is an adequate SVC gain for the 500kV Grajaú bus.

5.4. Report on the Use of the PoC Power Flow Program

Once a given scenario of load and generation increase is defined, the Point of Collapse Power Flow program can find in a direct manner the maximum loadability point, where the power flow Jacobian matrix becomes singular. The solution time needed to find the PoC (running in a modern UNIX Workstation) for the 1663-bus system may be as high as 20 times that needed for a single load flow solution. The results obtained were very precise and will be shown in conjunction with the modal analysis in the next section. This algorithm needs, however, to be made faster so as to be more effectively used in multiple scenario situations and minimum distance to collapse studies [15].

Reference [30] points out that Point of Collapse power flows, apart from being computationally very expensive, depend greatly on a good initial guess of the desired saddle-node bifurcation point. Otherwise, the method may diverge or converge to an undesired saddle-node bifurcation point. We had sometimes to use, out of convergence difficulties, the Continuation Power Flow program to obtain good enough initial values to the PoC program. These problems, which will certainly disappear with future enhancements, do not reduce the merits of this direct solution method.

5.5. Point of Collapse Results

The authors, like others [24], support the use of modal analysis mainly for highly stressed system conditions. The drawback of using modal analysis for medium-load conditions is the probable existence of not just one but several eigenvalues of equivalent moduli associated with the same system area. This fact impairs the practical assessment of the impact of alternative system changes to a base case operating point. This problem comes from the difficulty in keeping track of several voltage stability modes within a given area as the operating conditions are varied.

In the Point of Collapse Power Flow computer program, the scenario of load/generation increase is defined and kept fixed until a point of maximum loadability (singularity of the power flow Jacobian) is reached. The maximum loadability point represents a saddle-node bifurcation having a single zero eigenvalue associated with this load increase scenario. Attention is therefore focused on the bus voltage mode-shape, transfer function residues (or, alternatively, participation factors) associated with this single zero eigenvalue. Short-term remedial actions, system operation strategies and long-term system reinforcements can be ranked by these functions.

The base case condition (λ=-11.764) of Table 6 is very close (only 1.5% below) to the PoC condition. The residues, shown for the 500kV buses also in Table 6, provide a ranking list for the best buses to become voltage controlled ones (PV). A more powerful ranking information (though much more expensive) is provided from the zeros of $\frac{\Delta V(s)}{\Delta B_{sh}(s)}$, which are shown in Table 6.

The maximum system loadability (PoC) were calculated by the PoC Power Flow program and are given, in Table 6, in percentage loading above the base case condition. The generated MVAr at these buses, at the PoC solution, are presented in Table 6 and also have good correlation with the transfer function zero locations.

Note that the transfer function zero analysis at a given operating point is of practical use to area loadability studies, as long as no PV buses within the area reach Q-limits. When this happens, another transfer function zero analysis should be carried out.

Bus Converted to PV Type	kV Level	Zeros of $\Delta V/\Delta B_{sh}$	Residues of $\Delta V/\Delta B_{sh}$	PoC % Loading Above Base Case	Generated MVAr at Bus
none	-	-	-	1.5	-
Grajaú	500	-3.1072	0.30580	17.534	1551.8
Adrianópolis	500	-3.0510	0.28540	20.006	1772.47
Angra	500	-3.9419	0.22820	11.125	1123.52
C. Paulista	500	-3.7988	0.16100	12.624	1544.7
Taubaté	500	-5.5181	0.10380	5.8857	709.88
Tijuco Preto	500	-6.0962	0.07507	4.7813	623.35
Campinas	500	-7.3855	0.05644	3.7430	321.41
P. Caldas	500	-7.2441	0.05831	4.0176	351.55
Araraquara	500	-8.8341	0.02640	3.2631	224.51
Marimbondo	500	-10.6369	0.00417	2.8745	154.58
Jacarepaguá	345	-2.9985	†	18.4343	1718.0
Campos	345	-4.6254	†	6.7160	495.32

Table 6: *Critical Zeros of* $\frac{\Delta Vb^i(s)}{\Delta B^i_{sh}(s)}$ *for Base Case Power Flow with* λ=-11.764
The State Matrix is of Order 448

5.6. Voltage Stability Results Considering Induction Motor Loads

The RIO AREA, as described in Section 4, has a total load of 5000 MW which is distributed among 239 load buses.

The results presented on the previous sections were all related to the load model of Figure 1, which yields the state matrix $\left(-\mathrm{J}_{R}^{Q}\right)^{-1}$.

This section results were obtained including first-order induction motor models (Figure 2) to all the 239 load buses of the RIO AREA. The same motor parameters (typical values) were adopted for in all motors, except for their ratings, which were made compatible with the MVA values of the individual loads.

The voltage stability limit, for the load model of Figure 2, is reached at pull-out torque conditions where the critical eigenvalue becomes equal to zero (note that, for the previous model, the critical eigenvalue would go to infinity at the voltage collapse point).

Table 7 shows the system critical eigenvalues, obtained for the same power flow condition (base case), as the induction motors contents of the area load are gradually increased. The remaining part of the area load is treated as constant impedance.

Motor Contents in RIO AREA (% of Total Load)	Critical Eigenvalues	
60%	-14.242	λ_1
	-15.979	λ_2
	-16.612	λ_3
80%	-4.426	λ_1
	-6.632	λ_2
	-7.445	λ_3
85%	-1.103	λ_1
	-3.593	λ_2
	-4.479	λ_3
86%	-0.2416	λ_1
	-3.014	λ_2
	-3.825	λ_3
87%	+0.892	λ_1
	-2.416	λ_2
	-3.142	λ_3

Note: Bus with largest participation:
λ_1: Cachoeiro (#2617)
λ_2: A.Lage (#2611)
λ_3: J. Ne. (#2647)

Table 7: *Voltage Stability as a Function of Motor Load Contents in RIO AREA for Base Case Condition*

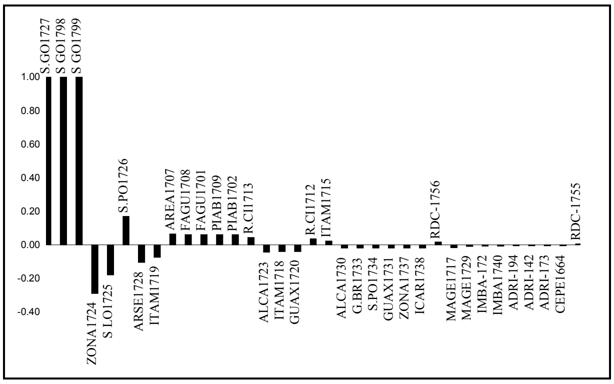


Figure 8: *Mode Shape for λ=-20 or -30*
(Denotes local behaviour only)

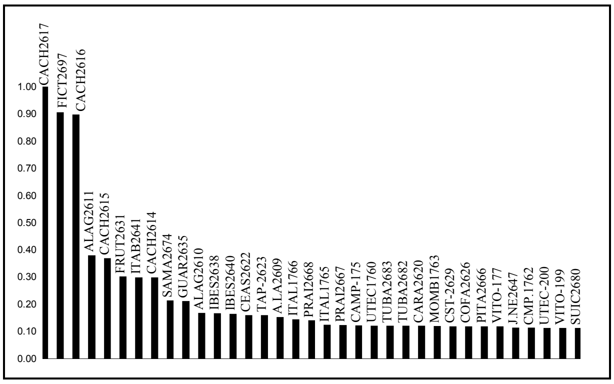


Figure 9: *Mode Shape for λ=-0.204*
(Denotes global area participation)

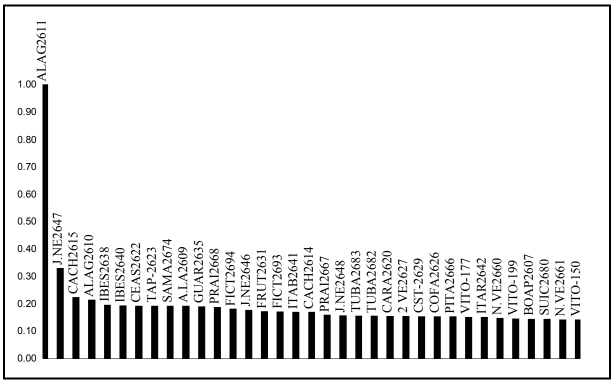


Figure 10: *Mode Shape for λ=-3.014*
(Denotes global area participation)

Most of the 239 system eigenvalues have values smaller then -20 and are related to local motor (or group of motors) behavior, as can be seen from the mode-shape of Figure 8. Only the 3 eigenvalues of Table 7 merit attention regarding voltage stability. Their mode-shapes (Figures 9 and 10) are similar to those obtained with the other load model (Figure 1). The area load stability limit occurs with 87% of motor content. The use of modal analysis results like these may be valuable in the determination of agregate load models for voltage stability.

6. NEW HORIZONS IN VOLTAGE STABILITY ANALYSIS

a) A conventional power flow program does not contain adequate generator, load and voltage control equipment models to produce realistic steady state solutions. Enhanced modeling is needed, as discussed in [2,3,5,7,8,23,25,26], to obtain better initial values for detailed large scale dynamic simulations, snapshots for modal analysis and more realistic point of collapse conditions.

b) The interest in the on-line assessment of voltage stability problems has been expressed in technical publications [1] and in international IEEE & CIGRÉ activities. Parallel eigensolution algorithms were applied to the electromechanical problem [33], and will next be applied to voltage stability. These parallel algorithms will render feasible the inclusion of on-line eigenanalysis functions, for voltage stability assessment purposes, in the Energy Management Systems of the future.

c) The application of probabilistic methods to the adequacy assessment of the bulk system transmission network is already well established in Brazil and abroad. Cepel and other Brazilian utilities have jointly developed a state-of-the-art generation/transmission reliability model. This package does not, however, directly consider the voltage stability margins of the system under analysis. Cepel is studying ways to add a voltage stability function to the generation/transmission reliability software for the purposes of adequacy studies. The action of emergency control aids, such as undervoltage load shedding to prevent voltage collapse [12], can be considered in the adequacy analysis.

The probabilistic approach described in [11], which introduces the supply interruption cost into the analysis, determines the tradeoff point between the risk of voltage instability and the cost of implementing preventive and corrective measures. The concepts expressed in [11] will be used to the further development of CEPEL's probabilistic tools. Parallel processing will play a major role in the integrated probabilistic tools of the future [34].

d) Existing optimum VAr allocation software are reported [9] to sometimes produce poor results from the voltage stability viewpoint. The strategy adopted in [9] was to determine preferred buses for shunt compensation based on modal analysis (participation factors of critical voltage modes) and then to split this compensation among these buses using an optimum VAr allocation software. Cepel is pursuing the idea of integrating the following existing tools into a single package: PoC and Continuation Power Flows, Optimum VAr Allocation [36], Optimum Power Flow and Modal Analysis.

The integration of all these programs into a generation/transmission reliability framework will yield a new software for the optimum probabilistic VAr allocation considering voltage stability margins.

e) Use of multivariable control theory, modal analysis included, in the design of secondary voltage and reactive power control strategies of large power grids [10]. Robust control techniques, if made computationally efficient to tackle large scale problems, will be valuable to this work.

f) Selective modal analysis [53] has always attracted great interest for its practically sound objective: finding only those system eigenvalues which are relevant to the problem under focus. The AESOPS algorithm was designed to only find the dominant oscillation modes in the torque-angle loops of the disturbed generator [45,46,42,47]. This concept has been recently extended in [40], which describes a most powerful algorithm to find the dominant poles of high order transfer functions between any pair of system variables. The results of [40] are related to the electromechanical stability problem, but the method has already been applied to the modal analysis of voltage stability. Once the system area under study is specified, the algorithm produces only the dominant modes of voltage collapse for this area. The further refinement of this class of algorithms will enhance the practical use of modal analysis and make it more amenable to non-expert engineers.

7. FINAL CONCLUSIONS

A range of methods aimed at voltage stability problems has been addressed, as well as their best forms of use. Some of the methods involving modal analysis are being presented for the first time in this paper. The results included indicate the power of the methods proposed.

The combination of Point of Collapse Power Flow and Modal Analysis proved to be synergetic as foreseen by others. The authors have obtained excellent results regarding system reinforcements or operative measures so as to maximize system loadability.

There is an urgent need for an advanced methodology to assess probabilistic voltage stability margins, to be applied both in expansion and operational planning stages [11]. Cepel is committed to the development of a comprehensive software package for this purpose.

The previous section, entitled "New Horizons on Voltage Stability Analysis", summarizes some of the future research and development work to be jointly carried out by Cepel, Eletrobrás and other Brazilian utilities.

8. ACKNOWLEDGMENTS

The authors would like to thank Dr. Fernando Alvarado for permission to use the academic versions of the PoC/Continuation Power Flow Programs in this research work.

9. REFERENCES

[1] **B. Gao, G.K. Morison and P. Kundur,** "Voltage Stability Evaluation Using Modal Analysis", *IEEE Trans. on Power Systems,* **Vol. 7, No. 4,** pp. 1529-1542, November 1992.

[2] **C.W. Taylor,** "Power System Voltage Stability", McGraw-Hill, 1993.

[3] "Voltage Stability of Power Systems: Concepts, Analytical Tools, and Industry Experience", *Report prepared by IEEE Working Group on Voltage Stability*, IEEE publication **90 TH 0358-2 PWR**, 1990.

[4] **CIGRÉ Task Force 38-02-10**, "Modeling of Voltage Collapse Including Dynamic Phenomena", *CIGRÉ Brochure*, No. 75, 1993 (summary in *Electra*, No. 147, pp. 71-77, April 1993).

[5] **Van Cutsem, T**., "Voltage Collapse Mechanisms: A Case Study", *Proceedings of Bulk Power System Voltage Phenomena - Voltage Stability and Security,* Aug. 1991.

[6] **W.W. Price, D.B. Klapper, N.W. Miller, A. Kurita and H. Okubo,** "A Multi-Faceted Approach to Power System Voltage Stability Analysis", paper presented in 1992 CIGRÉ Meeting, Paris, September 1992.

[7] **M.K. Pal,** "Voltage Stability Conditions Considering Load Characteristics", *1991 IEEE Summer Meeting,* paper **91 SM 490-3 PWRS**, San Diego, July 1991.

[8] **P.A. Löf, G. Andersson, D. Hill,** "Generator Modeling for Static Voltage Stability Studies", *Proceedings of PSCC*, 1993, pp. 923-929.

[9] **W. Xu, Y. Mansour,** "Application of VSTAB: A B.C. Hydro System Reinforcement Case", *presented at VSTAB user's group meeting*, Vancouver, July, 1993.

[10] **J. Florez, A. Tapia, R. Griado, J.M. Grijalba,** "Secondary Voltage Control Based on a Robust Multivariable PI Controller", *Proceedings of PSCC*, 1993, pp. 1011-1017.

[11] **X. Vieira F^o^., M.V.F. Pereira, P. Gomes, M.G. dos Santos and E. Nery**, "A Probabilistic Approach to Determine the Proximity of the Voltage Collapse Region", *1994 CIGRÉ Meeting, SC-38*, Paris, September 1994.

[12] **C.W. Taylor and others,** "Simulation and Implementation of Undervoltage Load Shedding for Pacific Northwest Voltage Stability", *1994 CIGRÉ General Meeting*, Paris, September, 1994.

[13] **I. Dobson, L. Lu,** "Voltage Collapse Precipitated by the Immediate Change in Stability When Generators Reactive Power Limits are Encountered", *IEEE Trans. on Circuits and Systems*, **Vol. 39 No. 9**, pp. 792-766, September 1992.

[14] **F.L. Alvarado, I. Dobson, Y. Hu,** "Computation of Closest Bifurcations in Power Systems", *1993 IEEE/PES Summer Meeting*, paper **93 SM 484-6 PWRS**, Vancouver, July 1993.

[15] **I. Dobson, L. Lu,** "New Methods for Computing a Closest Saddle-Node Bifurcation and Worst Case Load Power Margin for Voltage Collapse", *1992 IEEE/PES Summer Meeting,* paper **92 SM 587-6 PWRS***,* July 1992.

[16] **I. Dobson,** "Observations on the Geometry of Saddle-Node Bifurcation and Voltage Collapse in Electric Power Systems", *IEEE Trans. on Circuits and Systems*, Part 1, **39(3)**, pp. 240-243, March 1992.

[17] **V. Ajjarapu, C. Christy,** "The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis", *IEEE Trans. on Power Systems,* **No. 1,** pp. 416-423, February 1993.

[18] **C.A. Cañizares, F.L. Alvarado,** "Computational Experience with the Point of Collapse Method on Very Large AC/DC Systems", *in [23]*, pp. 103-111.

[19] **C.W. Taylor, B.L. Silverstein,** "Reliability Criteria for Voltage Stability", *Proceedings of IV SEPOPE*, Foz do Iguaçu, Brazil, May 1994.

[20] **Y. Mansour, E. Vaahedi, W. Xu, A. Chang,** "Power System Dynamics - Same Phenomena - New Challenges", *Proceedings of IV SEPOPE*, Foz do Iguaçu, Brazil, May 1994.

[21] **P. Kundur, B. Gao,** "Practical Considerations in Voltage Stability Assessment", *Proceedings of IV SEPOPE*, Foz do Iguaçu, Brazil, May 1994.

[22] **D.H. Hill,** "Modelling, Stability and Control of Voltage Behaviour in Power Supply", *Proceedings of IV SEPOPE*, Foz do Iguaçu, Brazil, May 1994.

[23] **L.H. Fink,** ed., "Proceedings: Bulk Power System Voltage Phenomena, Voltage Stability and Security", *ECC/NSF Workshop, Deep Creek Lake, MD*, August 1991, ECC Inc., 4400 Fair Lake Court, Fairfax, VA 22033-3899.

[24] **Y. Mansour, W. Xu, F. Alvarado, C. Rinzin,** "SVC Placement Using Critical Modes of Voltage Instability", *Proceedings of PICA Conference*, Phoenix, May 1993.

[25] **D.J. Hill**, "Nonlinear Dynamic Load Models with Recovery for Voltage Stability Studies", *1992 IEEE/PES Winter Meeting,* paper **92 WM 102-4 PWRS**, New York, January 1992.

[26] **W. Xu, Y. Mansour**, "Voltage Stability Analysis Using Generic Dynamic Load Models", *1993 IEEE/PES Winter Meeting,* paper **93 WM 185-9 PWRS,** Columbus, January 1993.

[27] **J. Deuse, M. Stubbe,** "Dynamic Simulation of Voltage Collapses", *IEEE Trans. on Power Systems,* **Vol. 8, N^o^. 3,** August 1993, pp. 894-904.

[28] **G.K. Morison, B. Gao, P. Kundur,** "Voltage Stability Analysis Using Static and Dynamic Approaches", *IEEE Trans. on Power Systems,* **Vol. 8, N^o^. 3,** August 1993, pp. 1159-1171.

[29] **P.A. Löf, G. Andersson, D.J. Hill,** "Voltage Stability Indices for Stressed Power Systems", *IEEE Trans. on Power Systems,* **Vol. 8, N^o^. 1,** February 1993, pp. 326-335.

[30] **H.D. Chiang, R. Jean-Jumean,** "Towards a Practical Performance Index for Predicting Voltage Collapse in Electric Power Systems", *1993 IEEE/PES Summer Meeting*, paper **93 SM 512-4 PWRS**, Vancouver, July 1993.

[31] **F.L. Alvarado,** "Voltage Stability, Bifurcation Parameters and Continuation Methods", *Proceedings of the IV SEPOPE,* Foz do Iguaçu, Brazil, May 1994.

[32] **N. Martins and L.T.G. Lima,** "Determination of Suitable Locations for Power System Stabilizers and Static VAr Compensators for Damping Electromechanical Oscillations in Large Scale Power Systems", *IEEE Trans. on Power Systems,*, Vol. **PWRS-5,** pp. 1455-1469, November 1990.

[33] **J.M. Campagnolo, N. Martins, J.L.R Pereira, L.T.G. Lima, H.J.C.P. Pinto and D.M. Falcão**, "Fast Small-Signal Stability Assessment Using Parallel Processing", *1993 IEEE/PES Summer Meeting*, paper **93 SM 481-2 PWRS***,* Vancouver, July 1993.

[34] **D.M. Falcão,** "Parallel and Distributed Processing in Power System Simulation and Control", *Proceedings of IV SEPOPE*, Foz do Iguaçu, Brazil, May 1994.

[35] **A.C.G. Melo, J.C.O. Mello and others,** "Reliability Evaluation of Large Scale Composite Power Systems", *Proceedings of III SEPOPE*, Belo Horizonte, Brazil, May 1992.

[36] **S. Granville, M.C.A. Lima, L.C. Lima, S. Prado,** "PLANVAR - An Optimization Software for VAr Sources Planning", *14th International Symposium on Mathematical Programming*, Amsterdam, August 1991.

[37] **T. Smed,** "Feasible Eigenvalue Sensitivity for Large Power Systems", *1992 IEEE/PES Winter Meeting,* paper **92 WM 171-9 PWRS**, New York, January 1992.

[38] **F.L. Alvarado,** "Bifurcations in Nonlinear Systems: Computational Issues", *Proceedings of ISCAS Conference*, New Orleans, Louisiana, pp. 922-925, May 1990.

[39] **F.L. Pagola, I.J. Pérez-Arriaga, G.C. Vergese,** "On Sensitivities, Residues and Participations: Applications to Oscillatory Stability Analysis and Control", *IEEE Trans. on Power Systems,* **Vol. PWRS-4***,* **N^o^. 1,** February 1989, pp. 278-285.

[40] **L.T.G. Lima, N. Martins, H.J.C.P. Pinto** "Mixed Real/Complex Factorization", *IEEE Trans. on Power Systems,* **Vol. 8, N^o^. 1**, pp. 302-308, February 1993.

[41] **B.T. Smith, J.M. Boyle, J. Dongarra and others,** "Matrix Eigensolution Routines: EISPACK Guide", *2nd. Edition, Springer-Verlag*, New York, 1976.

[42] **N. Martins, H.J.C.P. Pinto, L.T.G. Lima,** "Efficient Methods for Finding Transfer Function Zeros of Power Systems", *IEEE Trans. on Power Systems,* **Vol. PWRS-7,** pp. 1350-1361, August 1992.

[43] **N. Martins, L.T.G. Lima, H.J.C.P. Pinto,** "Computing Dominant Poles of Very High Order Transfer Functions: Application to Power System Dynamics and Control", *paper submitted to the 1995 IEEE/PES Winter Meeting*, New York, January 1995.

[44] **V.A. Venikov,** *Transient Phenomena on Electrical Power Systems,* Oxford Press, England, 1964.

[45] **R.T. Byerly, D.E. Sherman, R.J. Bennon**, "Frequency Domain Analysis of Low-Frequency Oscillations in Large Electric Power Systems", *EPRI EL-726 Project RP744-1, Final Report*, Part I, Palo Alto, CA, April 1978.

[46] **P.W. Sauer, O. Rajagolafan, M.A. Pai**, "An Explanation and Generalization of the AESOPS and PEALS Algorithms", *1990 Winter Meeting*, paper **90WM239-4 PWRS**, Atlanta, GA, February 1990.

[47] **D.M. Lam, H. Yee, B. Campbell,** "An Efficient Improvement of the AESOPS Algorithm for Power System Eigenvalue Calculation", *1994 IEEE/PES Winter Meeting*, paper **94WM250-1 PWRS**, New York, NY, January 1994.

[48] **N. Martins, L.T.G. Lima,** "Eigenvalue and Frequency Domain Analysis of Small-Signal Electromechanical Stability Problem", *IEEE Symposium on Application of Eigenanalysis and Frequency Domain Methods for System Dynamic Performance*, publication **90TH0292-3-PWR**, pp. 17-33, 1989.

[49] **N. Martins,** "Efficient Eigenvalue and Frequency Response Methods Applied to Power System Small-Signal Stability Studies", *IEEE Trans. on Power Systems*, **Vol. PWRS-1**, pp. 217-226, February 1986.

[50] **P. Kundur, G.J. Rogers, D.Y. Wong, L. Wang, M.G. Lauby,** "A Comprehensive Computer Program Package for Small-Signal Stability Analysis of Power Systems", *1990 IEEE Winter Meeting*, paper **90WM007-5 PWRS**, Atlanta, GA, February 1990.

[51] **B.T. Smith, J.M. Boyle, J. Dongarra, B. Garbow, Y. Ikebe, V.C. Klene, C.B. Moler,** *Matrix Eigensystem Routines: EISPACK Guide*, 2nd Edition, Springer-Verlag, New York, 1976.

[52] **A. Jennings,** *Matrix Computation for Engineers and Scientists*, John Wiley & Sons, 1977.

[53] **G.C. Verghese, I.J. Pérez-Arriaga, F.C. Schweppe,** "Selective Modal Analysis in Electric Power Systems", *EPRI EL-2830, Research Project 1764-8, Final Report*, January 1983.