Computing dominant poles of large second-order transfer functions

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1 Introduction

A transfer function of a large dynamical system often only has a small number of dominant poles compared to the number of state variables. Modal approximation techniques [1, 2, 3, 4] capture the dominant behavior of the system by projecting the state-space on the modes corresponding to the dominant poles. The computation of the dominant poles and modes requires specialized eigenvalue methods. In [5, 6, 7], algorithms were developed for the computation of dominant poles of single-input single-output (SISO) and multi-input multi-output (MIMO) transfer functions of large scale dynamical systems.

In this paper an efficient algorithm, the Quadratic Dominant Pole Algorithm (QDPA), for the computation of dominant poles of second-order transfer functions is described. Modal equivalents that are constructed by projecting the state-space matrices on the dominant left and right eigenspaces, preserve the structure of the original system. The dominant poles and modes can also be used to improve reduced-order models computed by rational Krylov based methods. For more details on the Quadratic Dominant Pole Algorithm, see [8, 9].

2 Second-order dynamical systems, transfer functions, and dominant poles

In this paper, the second-order dynamical systems $(M, C, K, \mathbf{b}, \mathbf{c}, d)$ are of the form

$$\begin{cases} M\ddot{\mathbf{x}}(t) + C\dot{\mathbf{x}}(t) + K\mathbf{x}(t) & = & \mathbf{b}u(t) \\ y(t) & = & \mathbf{c}^*\mathbf{x}(t) + du(t), \end{cases} \tag{1}$$

where $M, C, K \in \mathbb{R}^{n\times n}$ are the system matrices, $\mathbf{b}, \mathbf{c}, \mathbf{x}(t) \in \mathbb{R}^n$, $u(t), y(t), d \in \mathbb{R}$. The vectors $\mathbf{b}$ and $\mathbf{c}$ are called the input and output vector, respectively. The transfer function $H : \mathbb{C} \longrightarrow \mathbb{C}$ of (1) is defined as $H(s) = \mathbf{c}^*(s^2M + sC + K)^{-1}\mathbf{b} + d$. The poles of $H(s)$ are a subset of the eigenvalues $\lambda_i \in \mathbb{C}$ of the quadratic eigenvalue problem (QEP)

$$(\lambda_i^2 M + \lambda_i C + K)\mathbf{x}_i = 0, \quad \mathbf{y}_i^*(\lambda_i^2 M + \lambda_i C + K) = 0, \quad \mathbf{x}_i \neq 0, \quad \mathbf{y}_i \neq 0, \quad (i = 1, \ldots, 2n). \tag{2}$$

An eigentriplet $(\lambda_i, \mathbf{x}_i, \mathbf{y}_i)$ is composed of an eigenvalue λ_i and corresponding right and left eigenvectors $\mathbf{x}_i, \mathbf{y}_i \in \mathbb{C}^n$.

By transforming [10, Section 3.5] QEP (2) and system (1) to linear equivalents, the partial fraction representation becomes $H(s) = \mathbf{c}^* X(sI - \Lambda)^{-1}\Lambda Y^*\mathbf{b} = \sum_{i=1}^{2n} \frac{R_i}{s-\lambda_i}$, where $X = [\mathbf{x}_1, \ldots, \mathbf{x}_{2n}]$, $Y = [\mathbf{y}_1, \ldots, \mathbf{y}_{2n}]$, and $R_i = (\mathbf{c}^*\mathbf{x}_i)(\mathbf{y}_i^*\mathbf{b})\lambda_i$. The terms R_i are called the residues, and $\mathbf{x}_i$ and $\mathbf{y}_i$ are scaled so that $-\mathbf{y}_i^* K\mathbf{x}_i + \lambda_i^2\mathbf{y}_i^* M\mathbf{x}_i = 1$.

A pole λ_i of $H(s)$ with corresponding right and left eigenvectors $\mathbf{x}_i$ and $\mathbf{y}_i$ is called the *dominant* pole if $\widehat{R}_i = \frac{|R_i|}{\mathrm{Re}(\lambda_i)} > \widehat{R}_j$, for all $j \neq i$. More generally, a pole λ_i is called dominant if $|\widehat{R}_i|$ is not very small compared to $|\widehat{R}_j|$, for all $j \neq i$. This can also be seen in the corresponding Bode-plot, which is a plot of $|H(i\omega)|$ against $\omega \in \mathbb{R}$: peaks occur at frequencies ω close to the imaginary parts of the dominant poles of $H(s)$.

3 Quadratic Dominant Pole Algorithm

The poles of $H(s)$ are the $\lambda \in \mathbb{C}$ for which $\lim_{s\to\lambda} |H(s)| = \infty$ and hence $\lim_{s\to\lambda} 1/H(s) = 0$. In other words, the poles are the roots of $1/H(s)$ and a good candidate to find these roots is Newton's method: noting that $H'(s) = -\mathbf{c}^*(s^2M + sC + K)^{-1}(2sM + C)(s^2M + sC + K)^{-1}\mathbf{b}$ and starting with initial pole estimate s_0 gives the following scheme:

$$s_{k+1} = s_k + \frac{1}{H(s_k)}\frac{H^2(s_k)}{H'(s_k)} = s_k - \frac{\mathbf{c}^*\mathbf{v}}{\mathbf{w}^*(2s_kM + C)\mathbf{v}},$$

where $\mathbf{v} = (s_k^2M + s_kC + K)^{-1}\mathbf{b}$ and $\mathbf{w} = (s_k^2M + s_kC + K)^{-*}\mathbf{c}$.

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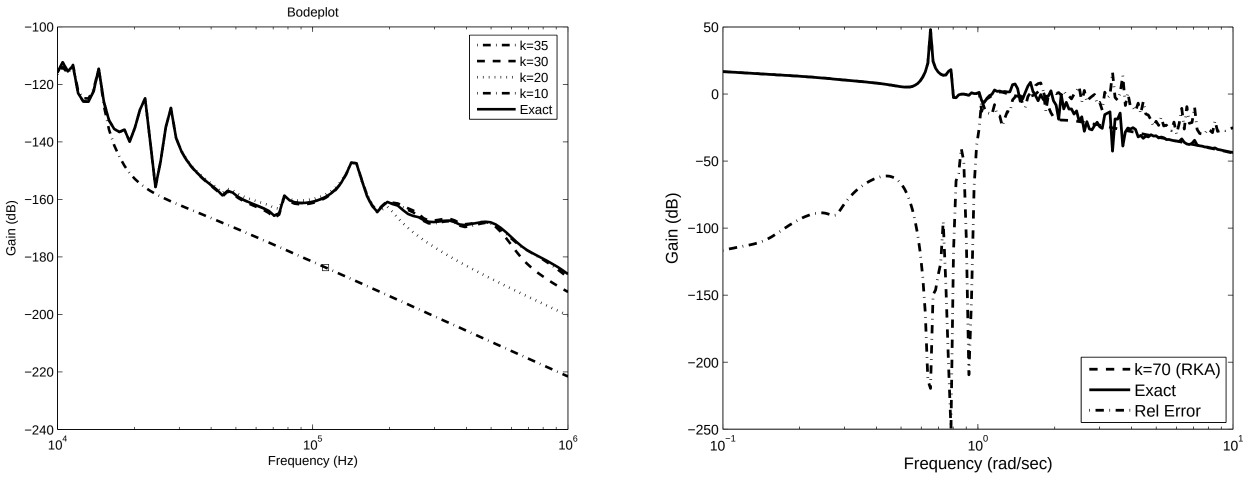

Fig. 1 Exact and reduced system transfer functions for the gyro (left) and vibrating body (right).

QDPA can be extended with subspace acceleration (keep search spaces for right and left eigenvectors) and a selection strategy (select the most dominant approximation every iteration) to improve global convergence to the most dominant poles, and deflation to avoid recomputation of already found poles. See [8] for more details.

4 Numerical results and conclusions

The left figure in Fig. 1 shows frequency response Bode plots of reduced order models based on 10, 20, 30, and 35 poles and corresponding eigenvectors, for a micro-mechanical gyro with $n = 17361$ degrees of freedom [11]. As can be seen from the matching of the peaks, QDPA finds the dominant poles.

The figure at the right in Fig. 1 shows the frequency response of a 70th order Second-Order Arnoldi [12] reduced model of vibrating body from sound radiation analysis ($n = 17611$ degrees of freedom), that was computed using the complex part $i\beta$ of dominant poles $\lambda = \alpha + i\beta$ (computed by QDPA) as interpolation points. This model is more accurate than reduced order models based on standard Krylov methods and matches the peaks up to $\omega = 1$ rad/s, because of use of shifts near the resonance frequencies.

The Quadratic Dominant Pole Algorithm (QDPA) is an efficient and effective method for the computation of dominant poles of second-order transfer functions. The dominant poles and corresponding left and right eigenvectors can be used to construct structure-preserving modal equivalents and to determine interpolation points for rational Krylov based model order reduction methods. QDPA can be generalized to MIMO systems and higher-order systems, and can be used for the computation of dominant zeros as well. For more details and results, see [8].

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