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MODAL ANALYSIS of INDUSTRIAL SYSTEM HARMONICS USING the S-DOMAIN APPROACH

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CEPEL

BACKGROUND

- Harmonic voltage performance of a system depends on the location of its poles and zeros mainly with respect to the characteristic harmonic frequencies
- Modal analysis finds poles, zeros and their respective sensitivities to system parameters
- Most effective parameter changes in order to reduce harmonic voltage levels

S-DOMAIN MODEL Y(*s*)

- The network is modeled by the Y(*s*) matrix, where *s* is the complex frequency given by $s=\sigma+j\omega$

- The Y(*s*) matrix is built just as the nodal admittance matrix Y($j\omega$)

- Replacing the purely imaginary frequency $j\omega$ for the complex frequency *s* is needed to perform modal analysis

- The first derivative of Y(*s*) with respect to the complex frequency *s* is also required

- This derivative can be easily obtained, following similar rules to those for building the Y(*s*) matrix

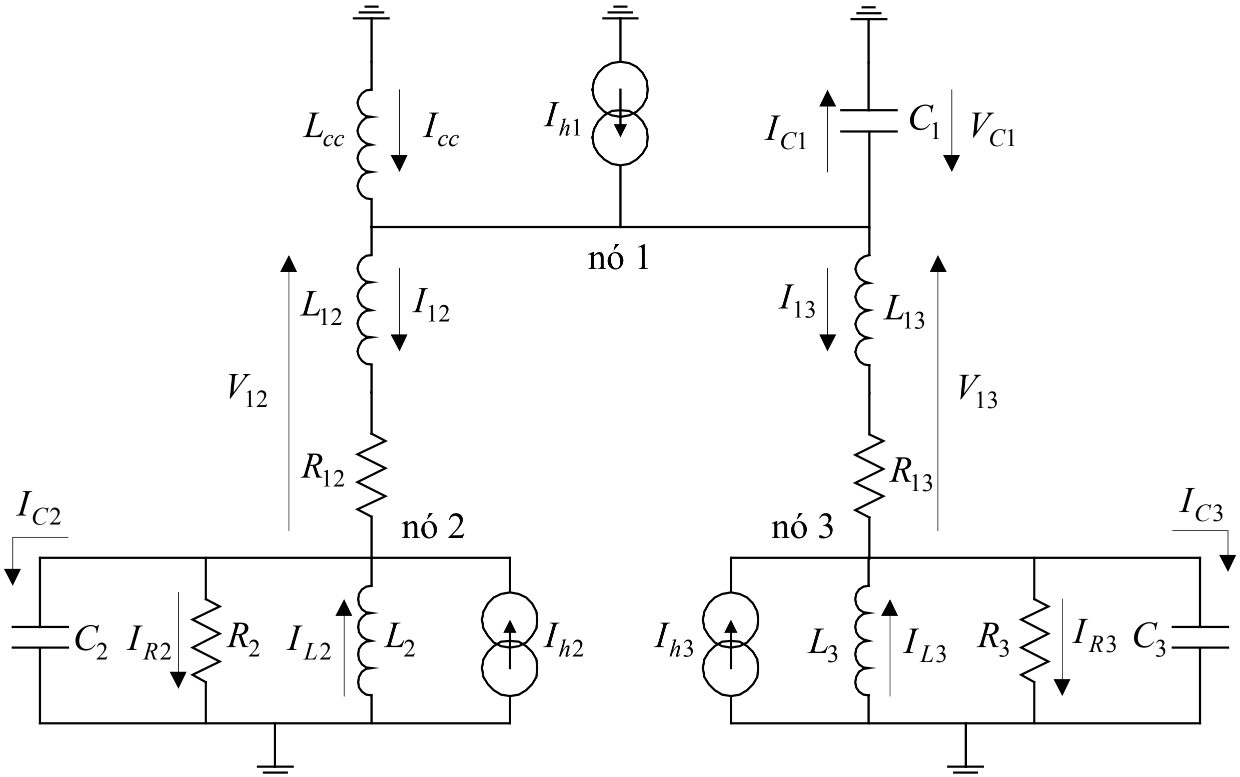
Nodal Admittance method (s-domain formulation)

$$\mathbf{Y}(s)=\begin{bmatrix} \frac{1}{s\,L_{cc}}+\frac{1}{R_{12}+s\,L_{12}}+s\,C_1+\frac{1}{R_{13}+s\,L_{13}} & -\frac{1}{R_{12}+s\,L_{12}} & -\frac{1}{R_{13}+s\,L_{13}} \\ -\frac{1}{R_{12}+s\,L_{12}} & \frac{1}{R_2}+\frac{1}{s\,L_2}+s\,C_2+\frac{1}{R_{12}+s\,L_{12}} & 0 \\ -\frac{1}{R_{13}+s\,L_{13}} & 0 & \frac{1}{R_3}+\frac{1}{s\,L_3}+s\,C_3+\frac{1}{R_{13}+s\,L_{13}} \end{bmatrix}$$

Network model:

$$\mathbf{Y}(s)\cdot\mathbf{V}=\mathbf{b}\cdot I_i$$

$$V_i=\mathbf{c}^t\cdot\mathbf{V}$$



Y(s) Derivative

$$\mathbf{Y}(s)=\begin{bmatrix} \frac{1}{s\,L_{cc}}+\frac{1}{R_{12}+s\,L_{12}}+s\,C_1+\frac{1}{R_{13}+s\,L_{13}} & -\frac{1}{R_{12}+s\,L_{12}} & -\frac{1}{R_{13}+s\,L_{13}} \\ -\frac{1}{R_{12}+s\,L_{12}} & \frac{1}{R_2}+\frac{1}{s\,L_2}+s\,C_2+\frac{1}{R_{12}+s\,L_{12}} & 0 \\ -\frac{1}{R_{13}+s\,L_{13}} & 0 & \frac{1}{R_3}+\frac{1}{s\,L_3}+s\,C_3+\frac{1}{R_{13}+s\,L_{13}} \end{bmatrix}$$

$$\frac{d\mathbf{Y}}{ds}=\begin{bmatrix} -\frac{1}{s^2\,L_{cc}}-\frac{L_{12}}{\left(R_{12}+s\,L_{12}\right)^2}+C_1-\frac{L_{13}}{\left(R_{13}+s\,L_{13}\right)^2} & \frac{L_{12}}{\left(R_{12}+s\,L_{12}\right)^2} & \frac{L_{13}}{\left(R_{13}+s\,L_{13}\right)^2} \\ \frac{L_{12}}{\left(R_{12}+s\,L_{12}\right)^2} & -\frac{1}{s^2\,L_2}+C_2-\frac{L_{12}}{\left(R_{12}+s\,L_{12}\right)^2} & 0 \\ \frac{L_{13}}{\left(R_{13}+s\,L_{13}\right)^2} & 0 & -\frac{1}{s^2\,L_3}+C_3-\frac{L_{13}}{\left(R_{13}+s\,L_{13}\right)^2} \end{bmatrix}$$

Linear analysis using s-domain formulation

s-Domain formulation:

$$\mathbf{Y}(s) \cdot \mathbf{x}(s) = \mathbf{b} \cdot u(s)$$

$$y(s) = \mathbf{c}^t \cdot \mathbf{x}(s)$$

Transfer function:

$$G(s) = \frac{y(s)}{u(s)} = \mathbf{c}^t \cdot \left[\mathbf{Y}(s)\right]^{-1} \cdot \mathbf{b}$$

Frequency response:

$$G(j\,\omega)$$

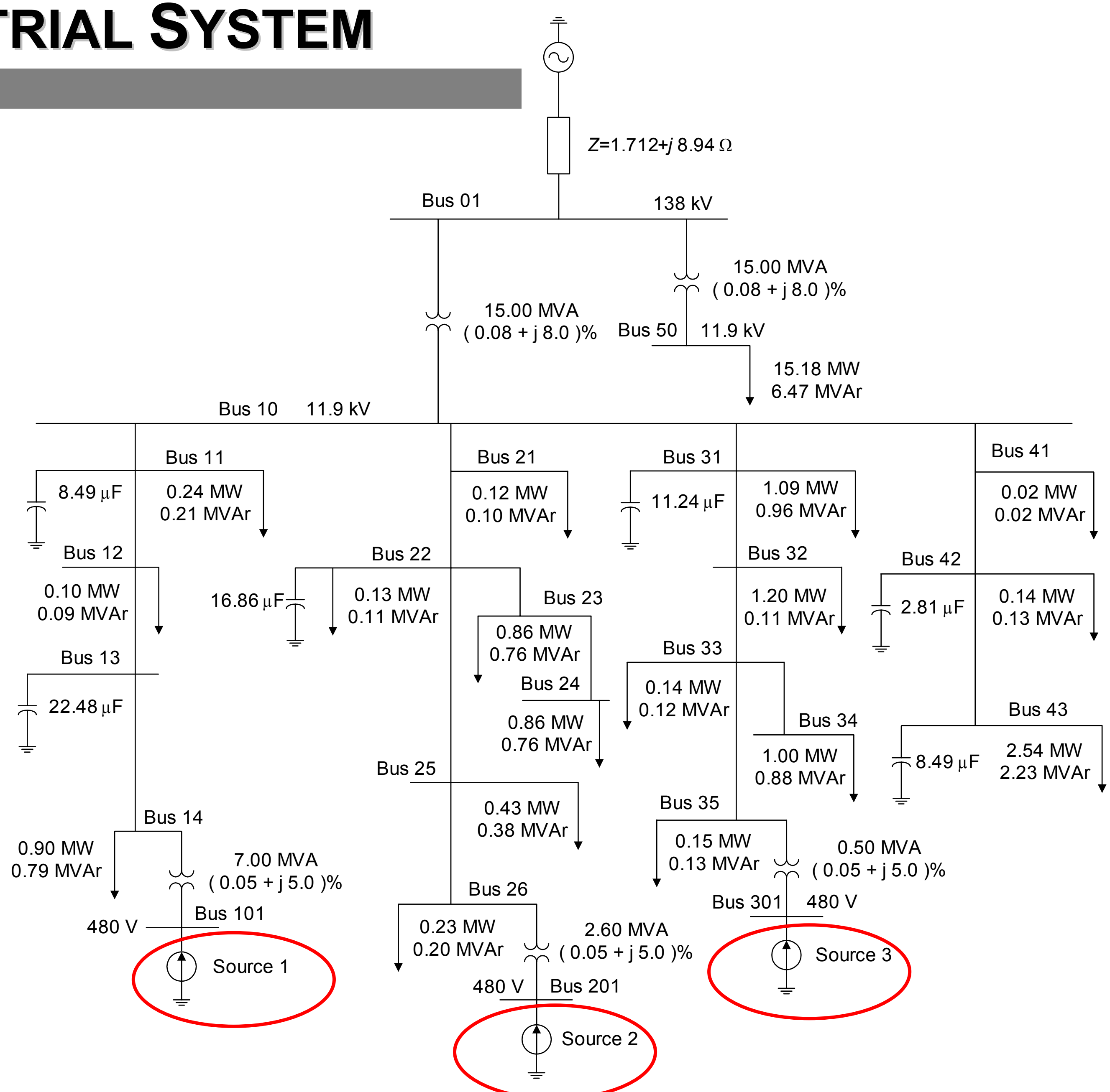
Modal analysis:

Pole: $\det\left[\mathbf{Y}(\lambda_i)\right] = 0$ $\qquad G(\lambda_i) \to \infty$

Zero: $G(z_i) = 0$

Tools: root-locus, sensitivities, modal time response, etc.

INDUSTRIAL SYSTEM



HARMONIC SOURCES CHARACTERISTICS

- Total current at fund. frequency drawn by the three sources = 8kA
- Source 1 contributes with 75%
- Source 2 contributes with 20%
- Source 3 contributes with 5%
- All three sources have the same harmonic contents

HARMONIC CURRENT COMPONENTS

h	3	5	7	9	11	13	15	17	19	21	23	25
I_h(%)	1.5	16.0	12.0	0.7	9.0	6.0	0.5	4.0	3.0	0.5	1.0	0.8

WEIGHTED IMPEDANCE MODULUS

- **Harmonic voltage (pu) at a bus *i* due the source 1, 2 and 3**

$$V_{i,1} = \mathbf{c}_i^t \, \mathbf{Y}(s)^{-1} \, \mathbf{b_1} \; 0.75 \; I'_h$$

$$V_{i,2} = \mathbf{c}_i^t \, \mathbf{Y}(s)^{-1} \, \mathbf{b}_2 \; 0.20 \; I'_h$$

$$V_{i,3} = \mathbf{c}_i^t \, \mathbf{Y}(s)^{-1} \, \mathbf{b}_3 \; 0.05 \; I'_h$$

- **Maximum harmonic voltage at a bus *i***

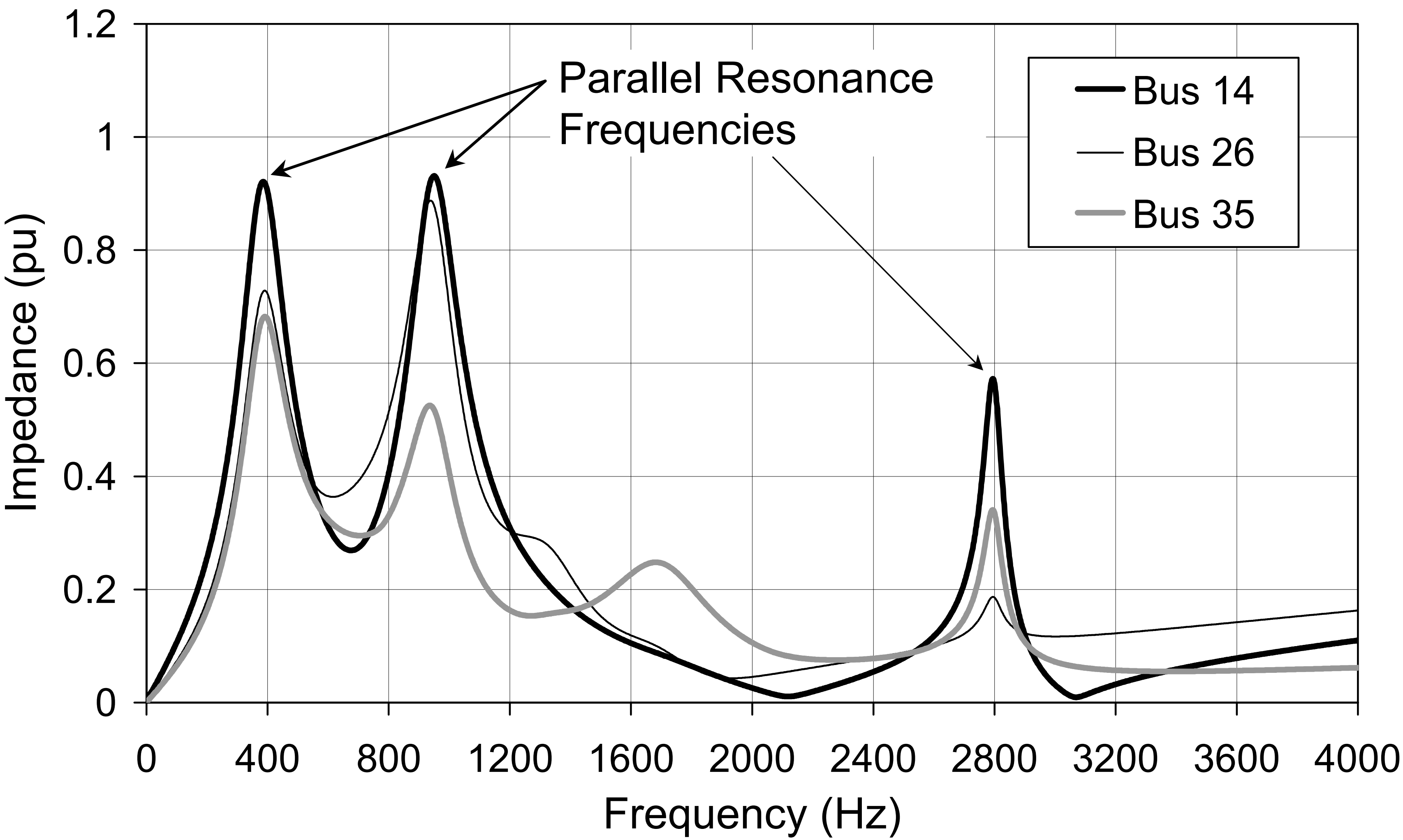
$$V_i = \sum_{j=1}^{3} \left| V_{i,j} \right| = \mathbf{c}_i^t \left| \mathbf{Y}(s)^{-1} \right| \mathbf{b} \; I'_h$$

$$\mathbf{b} = 0.75 \, \mathbf{b_1} + 0.20 \, \mathbf{b_2} + 0.05 \, \mathbf{b_3}$$

- **Weighted Impedance Modulus Definition**

$$\left| \overline{Z}_i \right| = \mathbf{c}_i^t \left| \mathbf{Y}(s)^{-1} \right| \mathbf{b}$$

WEIGHTED IMPEDANCE MODULUS



DOMINANT POLE ALGORITHM

Harmonic Impedance

$$z_{ij}(s) = \frac{V_i}{I_j} = \mathbf{c}^t\ \mathbf{Y}^{-1}(s)\,\mathbf{b}$$

System Poles

$$\frac{1}{z_{ij}(s)} = 0$$

Newton-Raphson

$$s^{(k+1)} = s^{(k)} + \left(\frac{dz_{ij}}{ds}\right)^{-1} z_{ij}$$

The Derivative of the Inverse of Y(*s*)

$$\frac{d\mathbf{Y}(s)^{-1}}{ds} = -\mathbf{Y}(s)^{-1}\,\frac{d\mathbf{Y}(s)}{ds}\,\mathbf{Y}(s)^{-1}$$

DOMINANT POLE ALGORITHM

Harmonic Impedance Derivative

$$\frac{dz_{ij}(s)}{ds} = -\mathbf{c}^t\ \mathbf{Y}(s)^{-1}\ \frac{d\mathbf{Y}(s)}{ds}\ \mathbf{Y}(s)^{-1}\ \mathbf{b}$$

Auxiliary Equations

$$\mathbf{Y}(s)^t\ \mathbf{w} = \mathbf{c} \qquad \mathbf{Y}(s)\ \mathbf{v} = \mathbf{b}$$

Newton-Raphson Algorithm

$$\begin{cases} z_{ij}(s) = \mathbf{c}^t\ \mathbf{v} \\ \dfrac{dz_{ij}(s)}{ds} = -\mathbf{w}^t\ \dfrac{d\mathbf{Y}(s)}{ds}\ \mathbf{v} \\ s^{(k+1)} = s^{(k)} - \dfrac{\mathbf{c}^t\ \mathbf{v}}{\mathbf{w}^t\ \dfrac{d\mathbf{Y}\left(s^{(k)}\right)}{ds}\ \mathbf{v}} \end{cases}$$

DOMINANT POLE ALGORITHM

CONVERGENCE PROCESS OF DOMINANT POLE ALGORITHM

Iter.	Pole 1 ($\lambda_1 = \sigma_1 + j\omega_1$)	Pole 2 ($\lambda_2 = \sigma_2 + j\omega_2$)
1	0.0000+j 2513.274	0.0000+j 6283.185
2	-495.9581+j 2385.159	-597.9614+j 5905.214
3	-459.9291+j 2400.484	-514.0175+j 5939.083
4	-459.9534+j 2400.514	-511.8524+j 5937.491
5	-459.9534+j 2400.514	-511.8543+j 5937.489
6	-	-511.8543+j 5937.489

Resonance Frequencies:

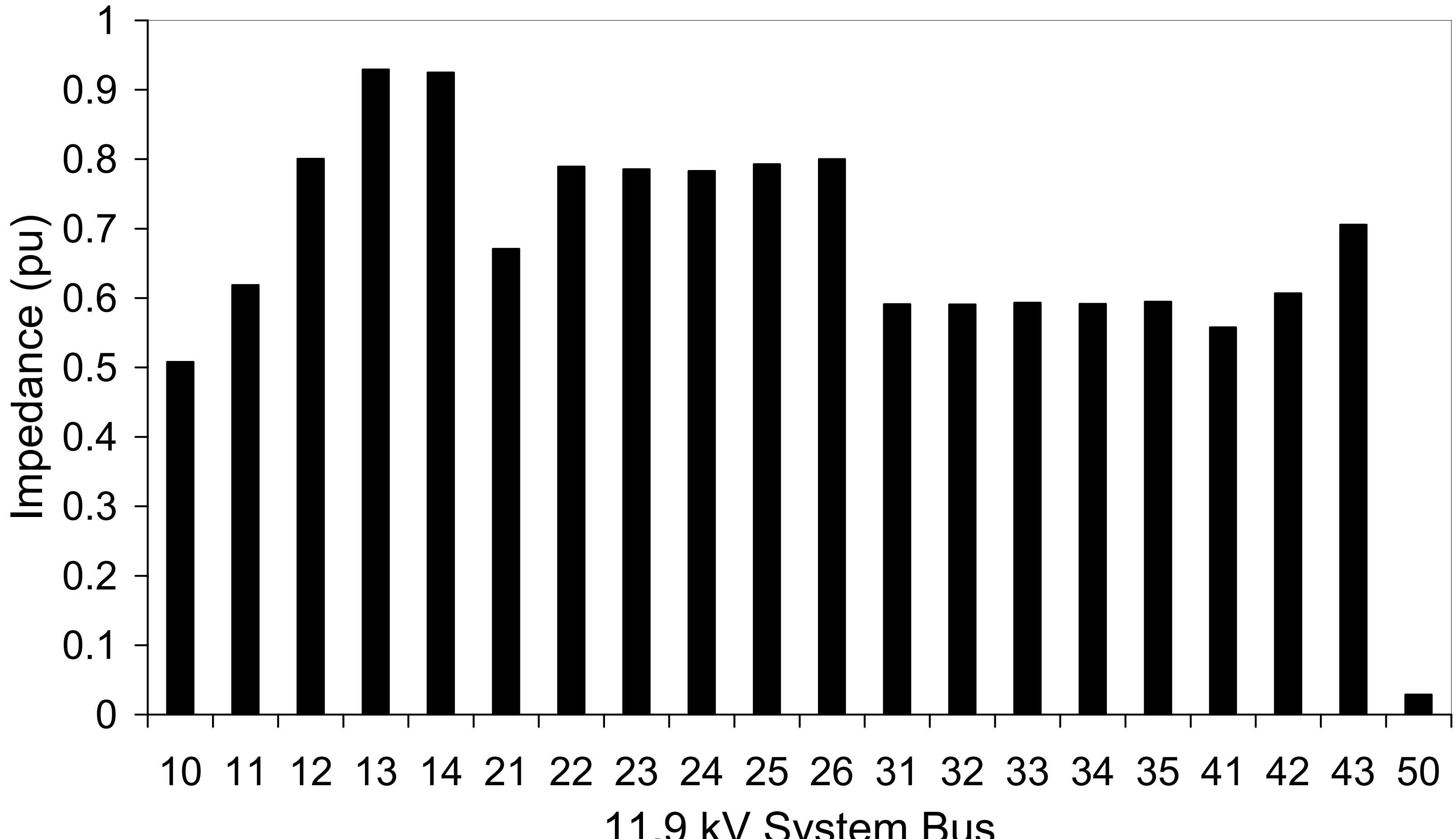
$$f_1 = \omega_1 /(2\pi) = 2400.514 /(2\pi) = 382.05\ Hz$$

$$f_2 = \omega_2 /(2\pi) = 5937.489 /(2\pi) = 944.98\ Hz$$

MODAL OBSERVABILITY INDEX

- **Definition**

$$MOI_i = \sqrt{\left|\overline{Z}_i(j\omega_1)\right| \times \left|\overline{Z}_i(j\omega_2)\right|}$$



Pole Weighted Impedance Function

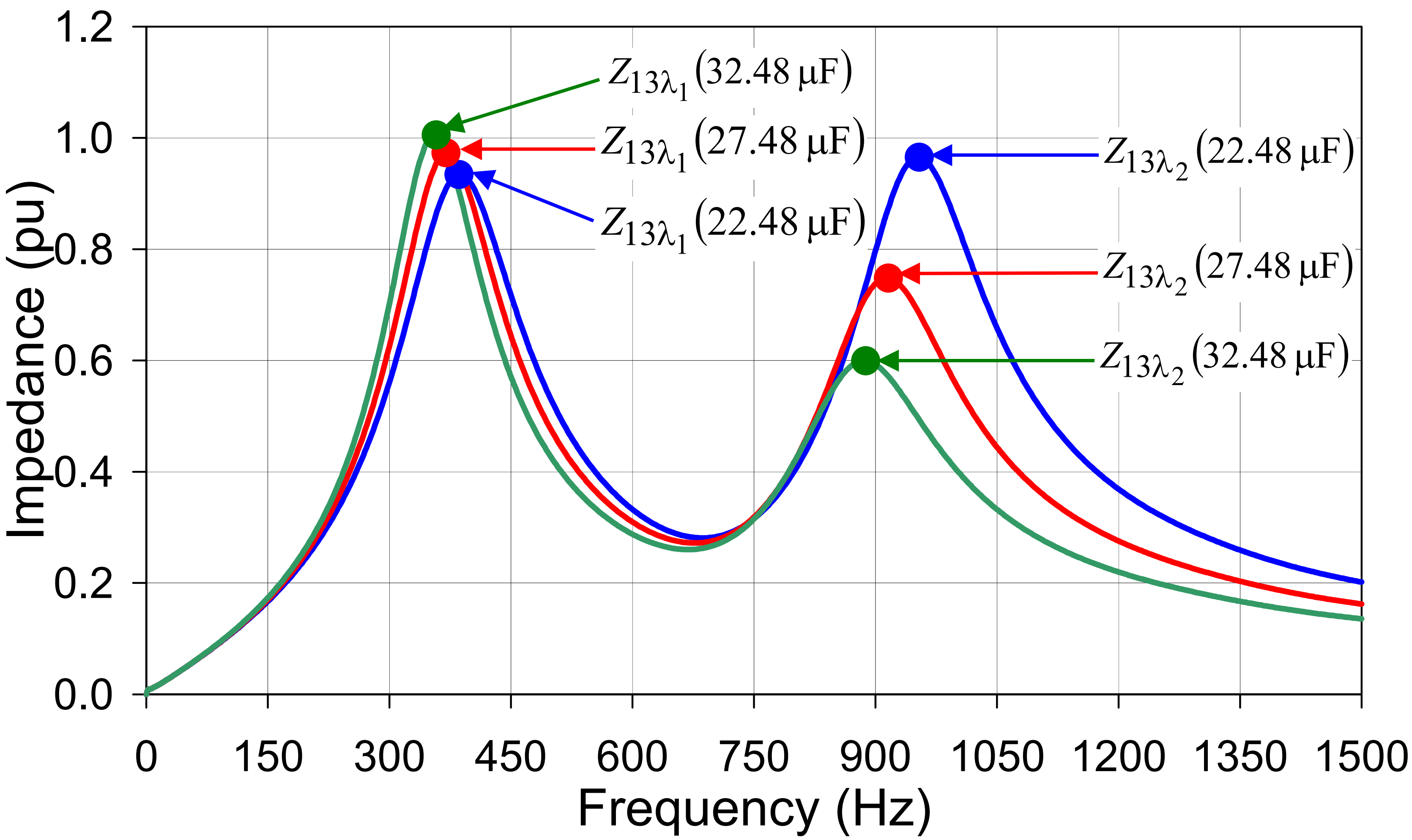
- **Definition: It is defined as the weighted impedance modulus seen from a bus *i* and calculated at all frequency values ω_k of a pole λ_k as an electric system parameter *p* varies.**

$$\overline{Z}_{i\lambda_k} = \left| \overline{Z}_i \left(j\omega_k \left(p \right) \right) \right|$$

- **Example: i = 13, k = 2 and p=C_{13}**

ΔC_{13} (μF)	C_{13} (μF)	$\lambda_2=\sigma_2+j\omega_2$ (s^{-1} and rad/s)	$f_2=\omega_2/(2\pi)$ (Hz)	$\overline{Z}_{13\lambda_2} = \left\|\overline{Z}_{13}(j\omega_2)\right\|$ (pu)
0.00	22.48	-511.85+j 5937.49	944.98	0.96
5.00	27.48	-535.79+j 5685.84	904.93	0.74
10.00	32.48	-558.74+j 5501.95	875.66	0.60
20.00	42.48	-598.00+j 5254.64	836.30	0.42

Pole Weighted Impedance Function



POLE SENSITIVITIES COEFFICIENTS

- **Pole Frequency (Parallel Resonant Frequency) Sensitivity:**

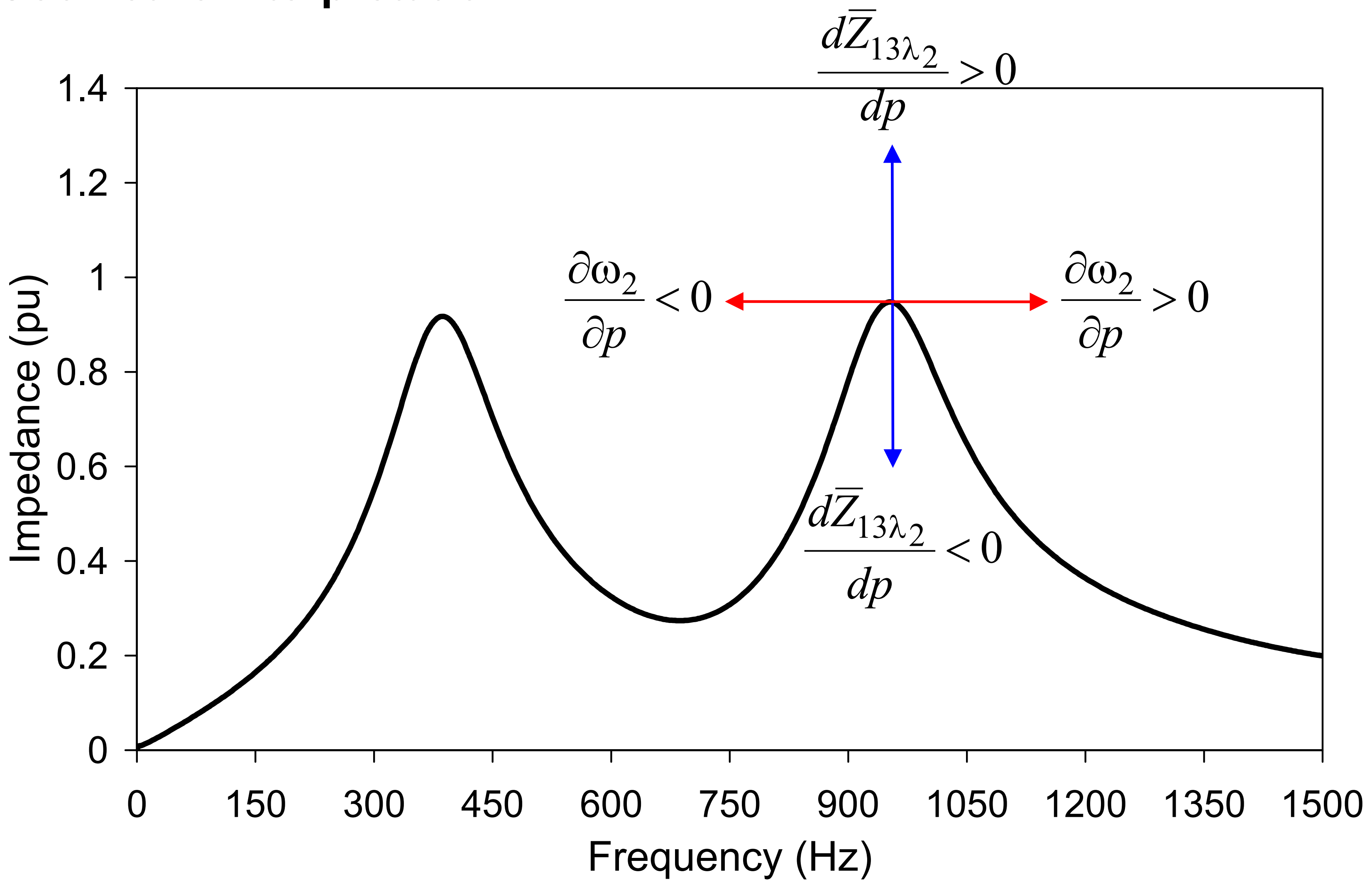
$$\frac{\partial \omega_k}{\partial p} = imag\left(\frac{\partial \lambda_k}{\partial p}\right) \qquad \frac{\partial \lambda_k}{\partial p} = -\frac{\mathbf{w}^t \dfrac{\partial \mathbf{Y}(\lambda_k, p)}{\partial p} \mathbf{v}}{\mathbf{w}^t \dfrac{\partial \mathbf{Y}(\lambda_k, p)}{\partial s} \mathbf{v}}$$

- **Pole Weighted Impedance Sensitivity:**

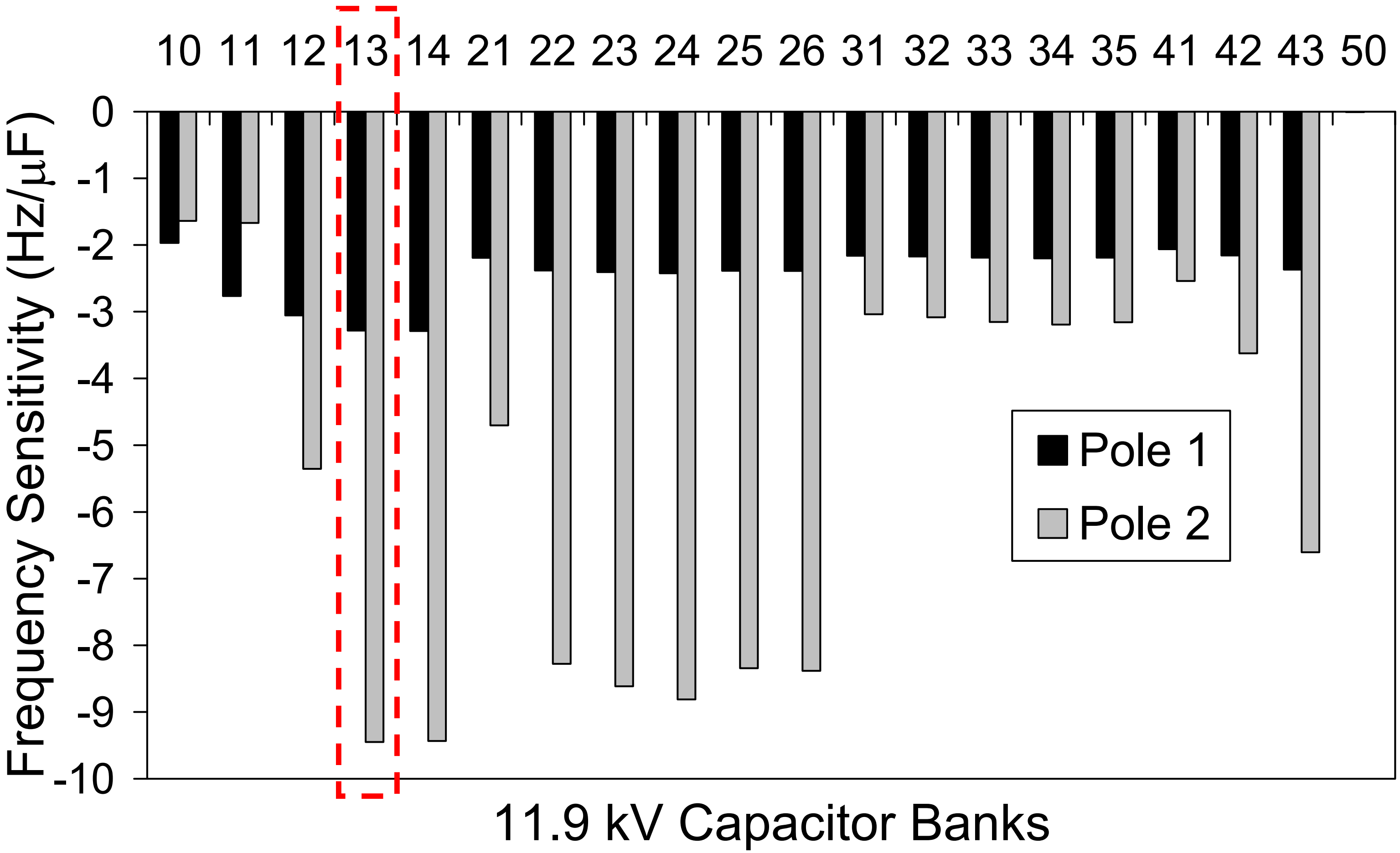
$$\left.\frac{d\overline{Z}_{i\lambda_k}}{dp}\right|_{p=p_0} = \frac{\left|\overline{Z}_i\left(j\omega_k\left(p_0 + \Delta p\right)\right)\right| - \left|\overline{Z}_i\left(j\omega_k\left(p_0 - \Delta p\right)\right)\right|}{2\Delta p}$$

Pole Sensitivities Coefficients

➢ **Geometric Interpretation**

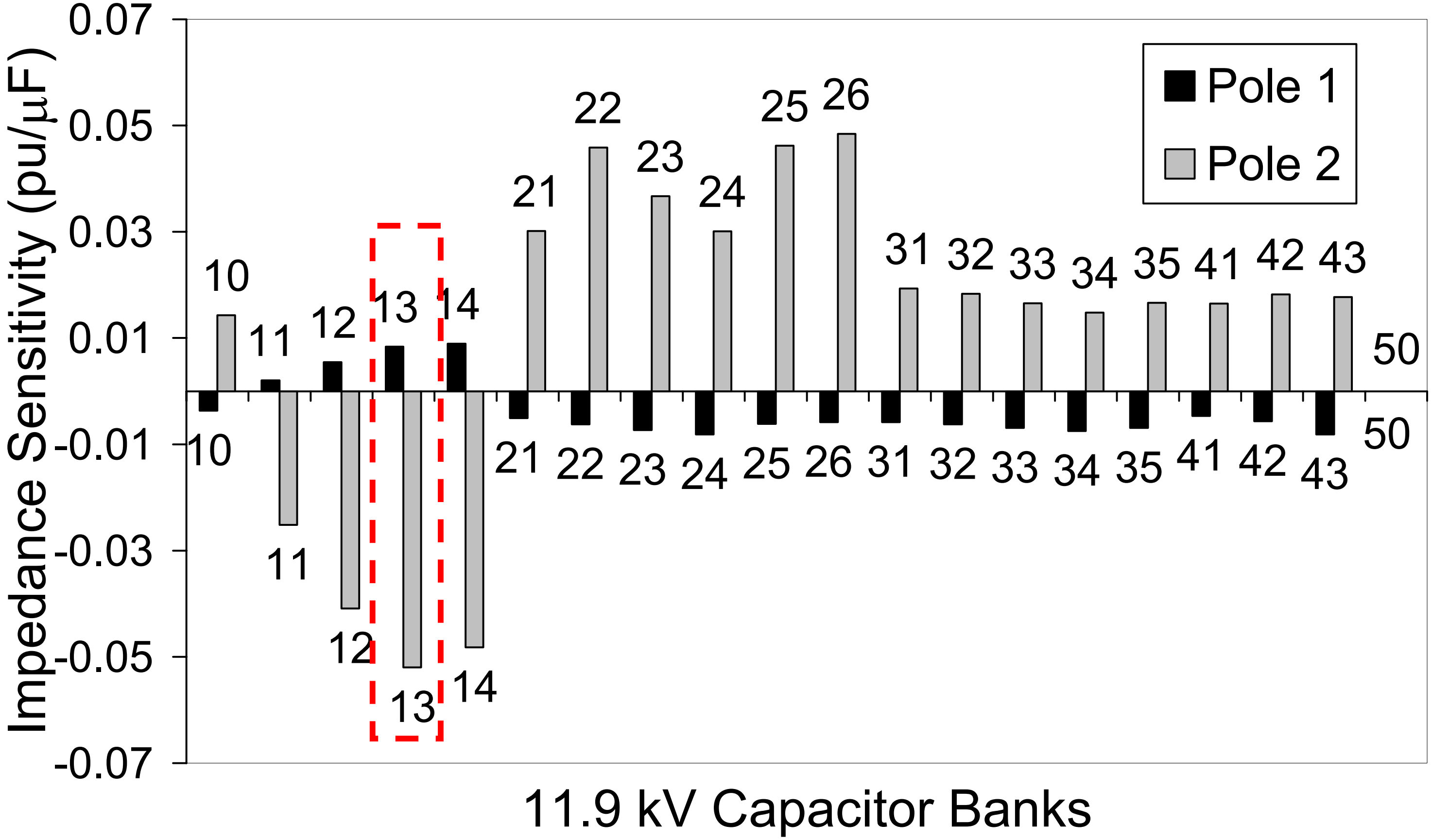


POLE FREQUENCY SENSITIVITIES



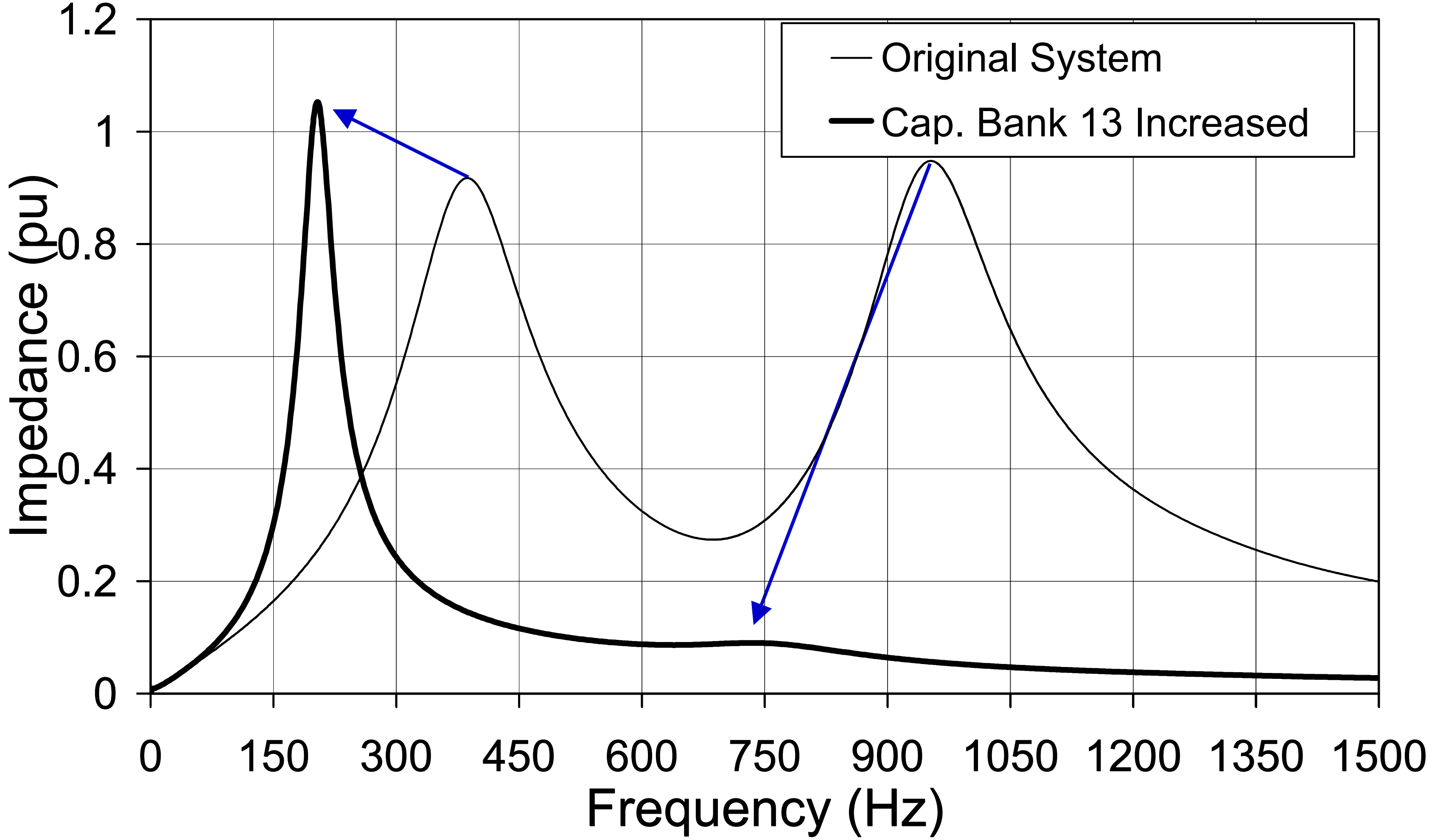
- **Main Conclusion: The increase of any capacitor bank will reduce the frequencies of the two poles. C_{13} is particularly effective.**

POLE WEIGHTED IMPEDANCE SENSITIVITIES



- **Main Conclusion: The weighted impedance modulus will decrease at the pole frequency 2 and increase a little at the pole frequency 1, if one or more of the capacitor banks 11, 12, 13 or 14 is increased.**

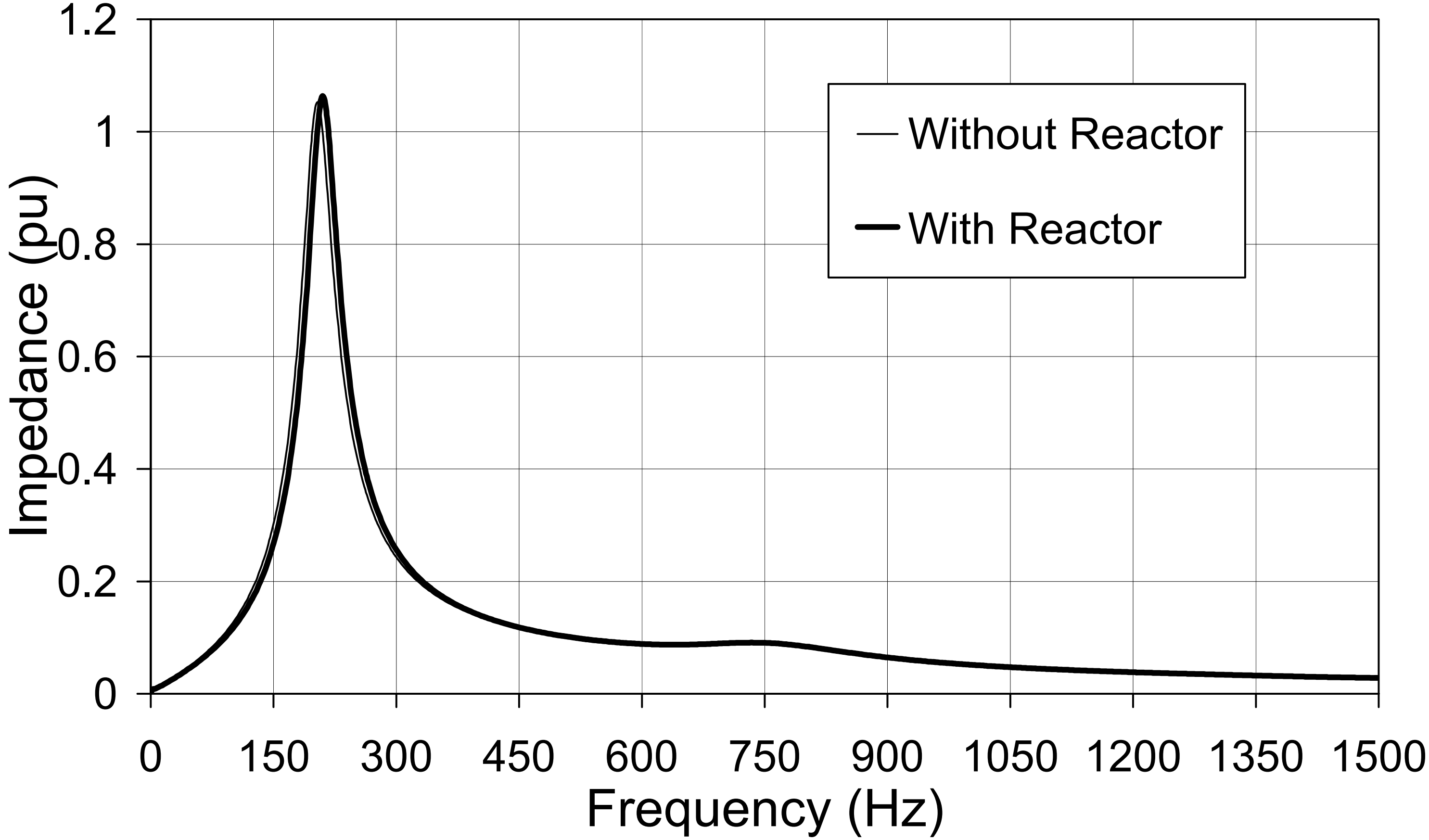
CAPACITOR BANK 13 INCREASED



- **Obs.: Additional capacitance ΔC_{13} = 125 μF**

SYSTEM OPERATING POINT MAINTAINING

- **The extra MVAr produced by the additional capacitor bank of 125 μ*F* must be compensated by a parallel reactor bank of the same power (56.59 *mH*)**



HARMONIC DISTORTION REDUCTIONS

HARMONIC VOLTAGE DISTORTION SPECTRUM AT BUS 13

Harmonic Order	Original System (%)	Proposed Solution (%)	Limits (%)
3	0.321	0.807	5
5	8.97	4.17	6
7	10.24	1.59	5
9	0.299	0.0683	1.5
11	2.58	0.809	3.5
13	2.16	0.538	3
15	0.399	0.0329	0.3
17	3.07	0.202	2
19	1.34	0.126	1.5
21	0.158	0.0181	0.2
23	0.245	0.0318	1.5
25	0.161	0.0228	1.5

THD = 4.6 %

THD Limit = 8 %

- **The Adopted limits are in accordance with IEC 1000-3-6-Electromagnetic Compatibility (EMC), Part 3, Section 6: "Assessment of Emission Limits for Distortion Loads in MV and HV Power Systems", First Edition, 1996-10**

Conclusions and Remarks

- This paper describes some results associated with the use of modal analysis to solve harmonic problems.
- This research is still in its initial stage and a lot of work must be done considering practical aspects of this methodology compared with the more traditional ones.
- Basically there are three forms of improving the harmonic performance of a system: Filtering harmonic currents, improving the performance of nonlinear loads and system modifications. This paper is a contribution for the third form.
- System modifications seems to be particularly suitable for reducing harmonic distortions in larger systems which have several and spread nonlinear loads.