AUGMENTED STATE-SPACE FORMULATION FOR THE STUDY OF ELECTRIC NETWORKS INCLUDING DISTRIBUTED-PARAMETER TRANSMISSION LINE MODELS

Leonardo T. G. Lima - UFF
Nelson Martins - CEPEL
Sandoval Carneiro Jr. - COPPE/UFRJ
Background

- Linear systems techniques
  - frequency response
  - eigenvalue/eigenvector analysis
  - sensitivities
  - transfer function residues, etc.

- State-space model

- Efficient large scale systems algorithms
Difficulties

- The construction of the state matrix for practical systems is not a simple task.
- Methods based on state matrix formulations present some limitations regarding network topology and not automatically deal with state variables redundancy.
- State-space representation of distributed-parameter models.
Descriptor System Approach

\[ T \dot{x} = Ax + Bu \]

\[ y = Cx \]

- Overcomes the computational difficulties associated with the state matrix method.
- Automatically deals with state variable redundancies
- Can be efficiently applied to large-scale networks of any topology
Objectives

- Application of the Descriptor System Approach to study electric networks including distributed-parameter transmission lines

- Establish the grounds for the determination of dynamic equivalents
System Model

Network Model

\[
\begin{align*}
\frac{d}{dt} \mathbf{x}(t) & = A \mathbf{x}(t) + B \mathbf{i}_{\text{nodal}}(t) \\
\frac{d}{dt} \mathbf{v}(t) & = C \mathbf{x}(t) + D \mathbf{i}_{\text{nodal}}(t)
\end{align*}
\]

Impedance as a transfer function

\[
Z_{kk}(s) = \frac{v_k(s)}{i_k(s)} = C_k \left( s \mathbf{T-A} \right) \mathbf{B}_k
\]
System Model

Each network component is modeled as a descriptor system with terminal voltages as inputs and current injections as outputs

\[ T_i \dot{x}_i = A_i x_i + L_i v \]

\[ i_i = M_i x_i + N_i v \]

System integration through Kirchoff current law applied to each node of the network
Distributed Parameter Model

\[ V_k(s) = B_k(s) + Z_c(s) I_k(s) \]
\[ B_k(s) = A_1(s) \left[ V_m(s) + Z_c(s) I_m(s) \right] \]
\[ V_m(s) = B_m(s) + Z_c(s) I_m(s) \]
\[ B_m(s) = A_1(s) \left[ V_k(s) + Z_c(s) I_k(s) \right] \]
Distributed Parameter Model

\[
e^{-st} = \frac{2 - \tau s + \left(-\tau s\right)^2}{2!} + \frac{\left(-\tau s\right)^3}{3!} \ldots
\]

\[
\frac{2 + \tau s + \left(\tau s\right)^2}{2!} + \frac{\left(\tau s\right)^3}{3!} \ldots
\]

\[
\text{Padé approximation}
\]
Test System
Results

\[ \frac{V_{a3}(s)}{I_{a3}(s)} \]

Magnitude (dB)

frequency (Hz)

--- Padé=10  --- Padé=20  --- Padé=30  ---- ATP
Results

\[ \frac{V_{b3}(s)}{I_{a3}(s)} \]

Magnitude (dB)

frequency (Hz)

- - - - - Padé=10  - - - - Padé=20  - - - - Padé=30  ——— ATP
$\frac{V_{a3}(s)}{I_{a3}(s)}$

Reduced Order Equivalent

- ATP
- Order=53
- Order=69
- Order=83
Reduced Order Equivalent

\[
\frac{V_{c3}(s)}{I_{a3}(s)}
\]
Conclusions

State-space based formulations obtain the same frequency domain results as the conventional methods based on nodal formulation and more:

- Modal analysis techniques can be used to obtain poles, zeros, transfer function residues, sensitivities, etc.
- Reduced order dynamic equivalents obtained through use of transfer function residues
Conclusions

Descriptor system approach allows:

- Simple and efficient computational implementation
- Ability to model systems of any topology and containing state variable redundancies
- Applicability to large-scale networks, due to the very sparse matrices involved and the availability of powerful sparse eigensolution algorithms applied to descriptor systems