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AUGMENTED STATE-SPACE FORMULATION FOR THE STUDY OF ELECTRIC NETWORKS INCLUDING DISTRIBUTED-PARAMETER TRANSMISSION LINE MODELS

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Background

- Linear systems techniques
  - frequency response
  - eigenvalue/eigenvector analysis
  - sensitivities
  - transfer function residues, etc.
- State-space model
- Efficient large scale systems algorithms

Difficulties

- The construction of the state matrix for practical systems is not a simple task
- Methods based on state matrix formulations present some limitations regarding network topology and not automatically deal with state variables redundancy
- State-space representation of distributed-parameter models

Descriptor System Approach

$$\mathbf{T}\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\,\mathbf{x}$$

- Overcomes the computational difficulties associated with the state matrix method.
- Automatically deals with state variable redundancies
- Can be efficiently applied to large-scale networks of any topology

Objectives

- Application of the Descriptor System Approach to study electric networks including distributed-parameters transmission lines

- Establish the grounds for the determination of dynamic equivalents

System Model

- Network Model

$$\begin{bmatrix} \begin{matrix} T_1 & & \\ & T_2 & \\ & & \ddots & \\ & & & T_n \end{matrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \frac{d}{dt}\mathbf{x}(t) \\ \frac{d}{dt}\mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \begin{matrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_n \end{matrix} & \begin{matrix} L_1 \\ L_2 \\ \vdots \\ L_n \end{matrix} \\ \begin{matrix} M_1 & M_2 & \cdots & M_n \end{matrix} & N \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \mathbf{i}_{nodal}(t)$$

- Impedance as a transfer function

$$z_{kk}(s) = \frac{v_k(s)}{i_k(s)} = \mathbf{C}_k \left(s\,\mathbf{T}\text{-}\mathbf{A}\right)\mathbf{B}_k$$

System Model

- Each network component is modeled as a descriptor system with terminal voltages as inputs and current injections as outputs

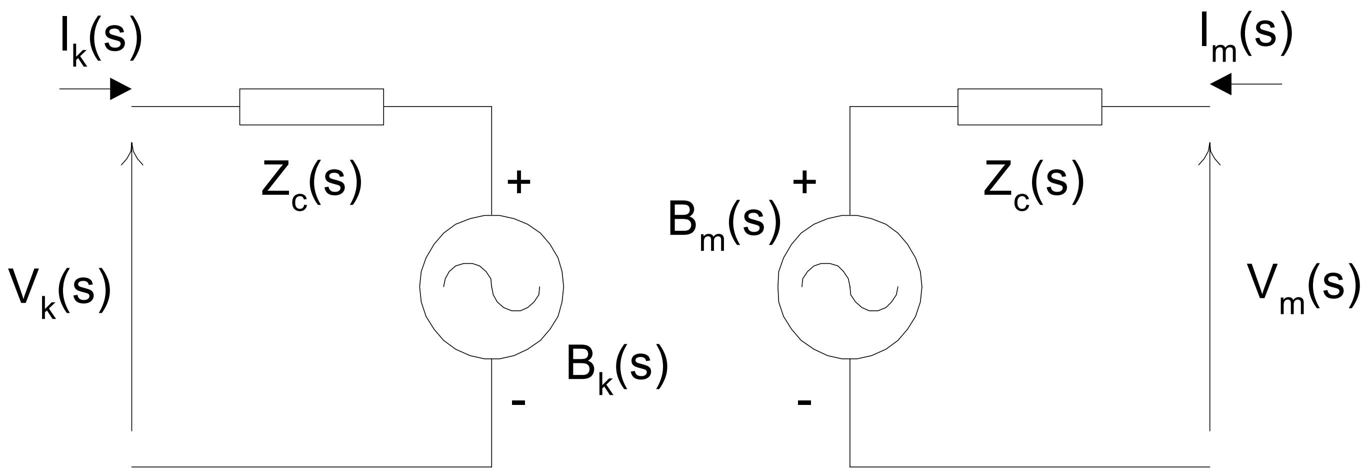
$$\mathbf{T}_i \dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + \mathbf{L}_i \mathbf{v}$$

$$\mathbf{i}_i = \mathbf{M}_i \mathbf{x}_i + \mathbf{N}_i \mathbf{v}$$

- System integration through Kirchoff current law applied to each node of the network

Distributed Parameter Model

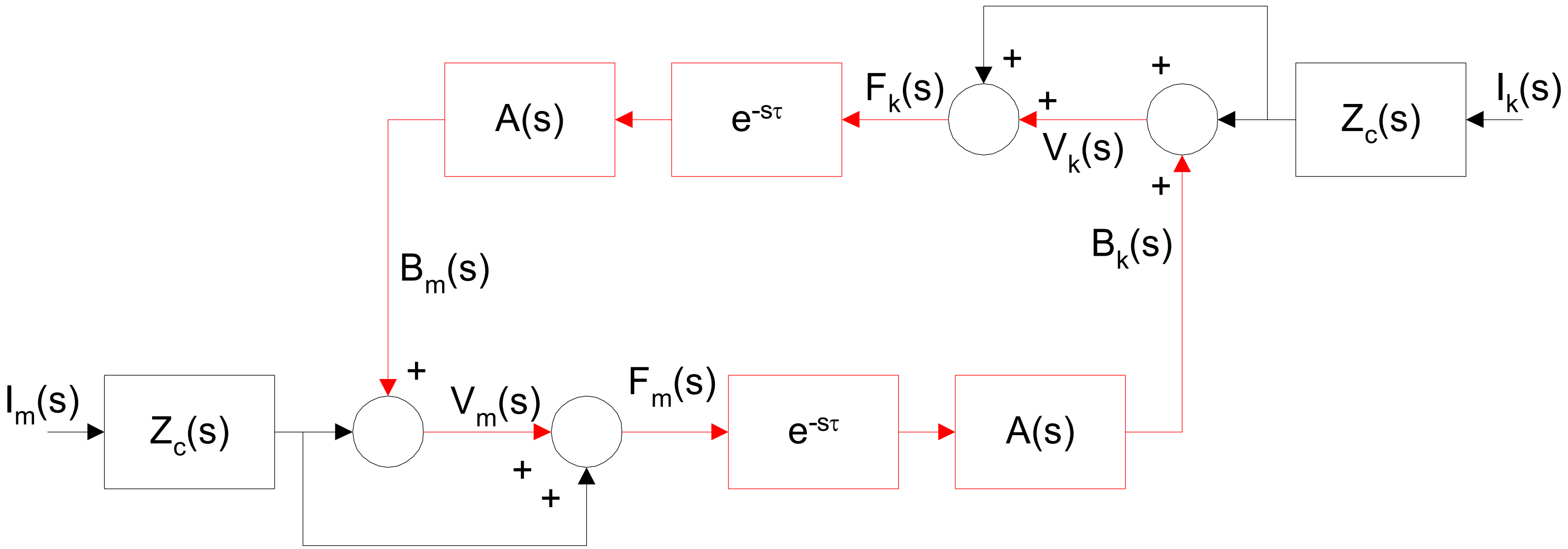
- Equivalent System



$$V_k(s) = B_k(s) + Z_C(s) I_k(s) \qquad B_k(s) = A_1(s)\left[V_m(s) + Z_C(s) I_m(s)\right]$$

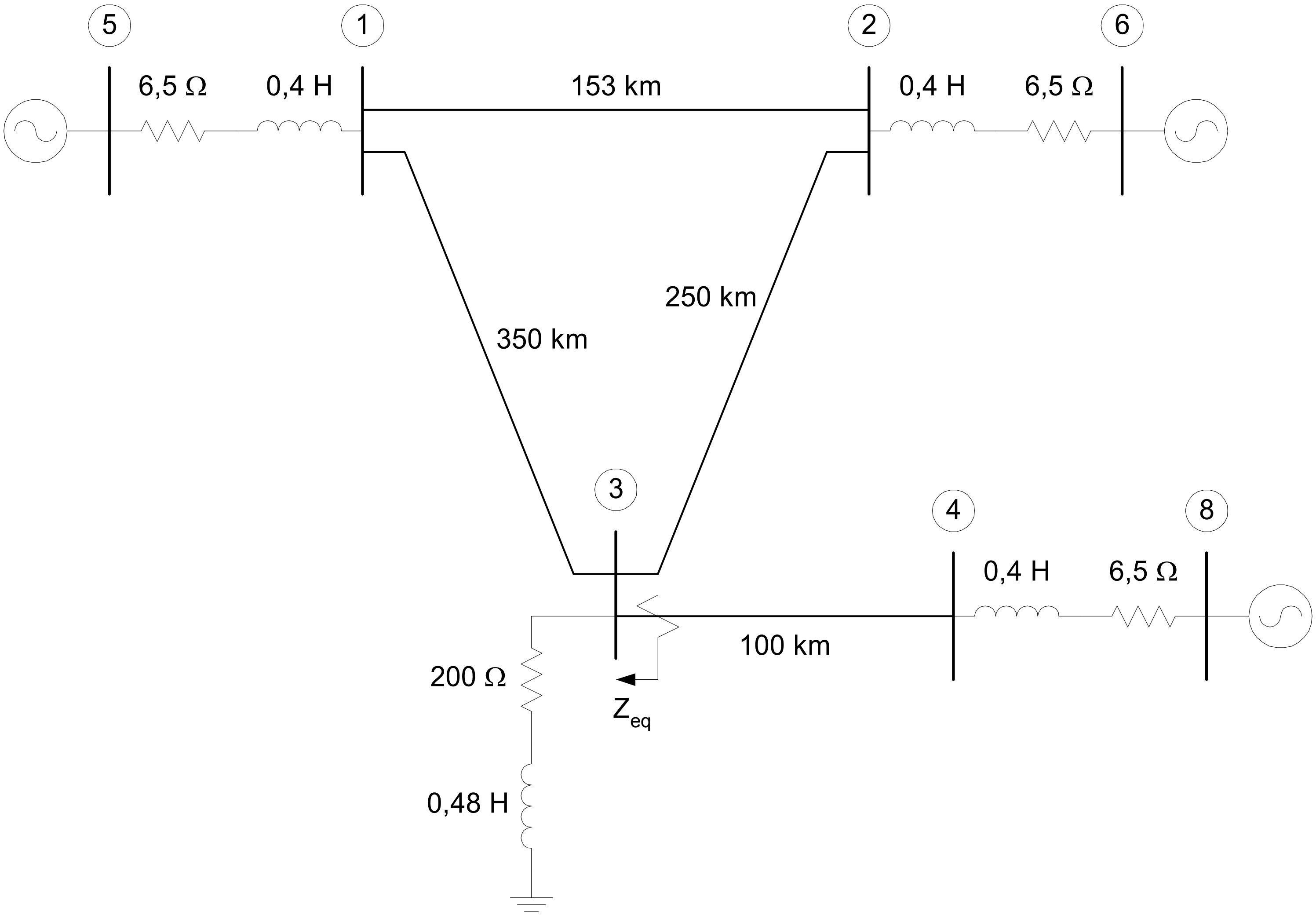
$$V_m(s) = B_m(s) + Z_C(s) I_m(s) \qquad B_m(s) = A_1(s)\left[V_k(s) + Z_C(s) I_k(s)\right]$$

Distributed Parameter Model



- Padé approximation $e^{-st} = \dfrac{2 - \tau s + \dfrac{(-\tau s)^2}{2!} + \dfrac{(-\tau s)^3}{3!} \cdots}{2 + \tau s + \dfrac{(\tau s)^2}{2!} + \dfrac{(\tau s)^3}{3!} \cdots}$

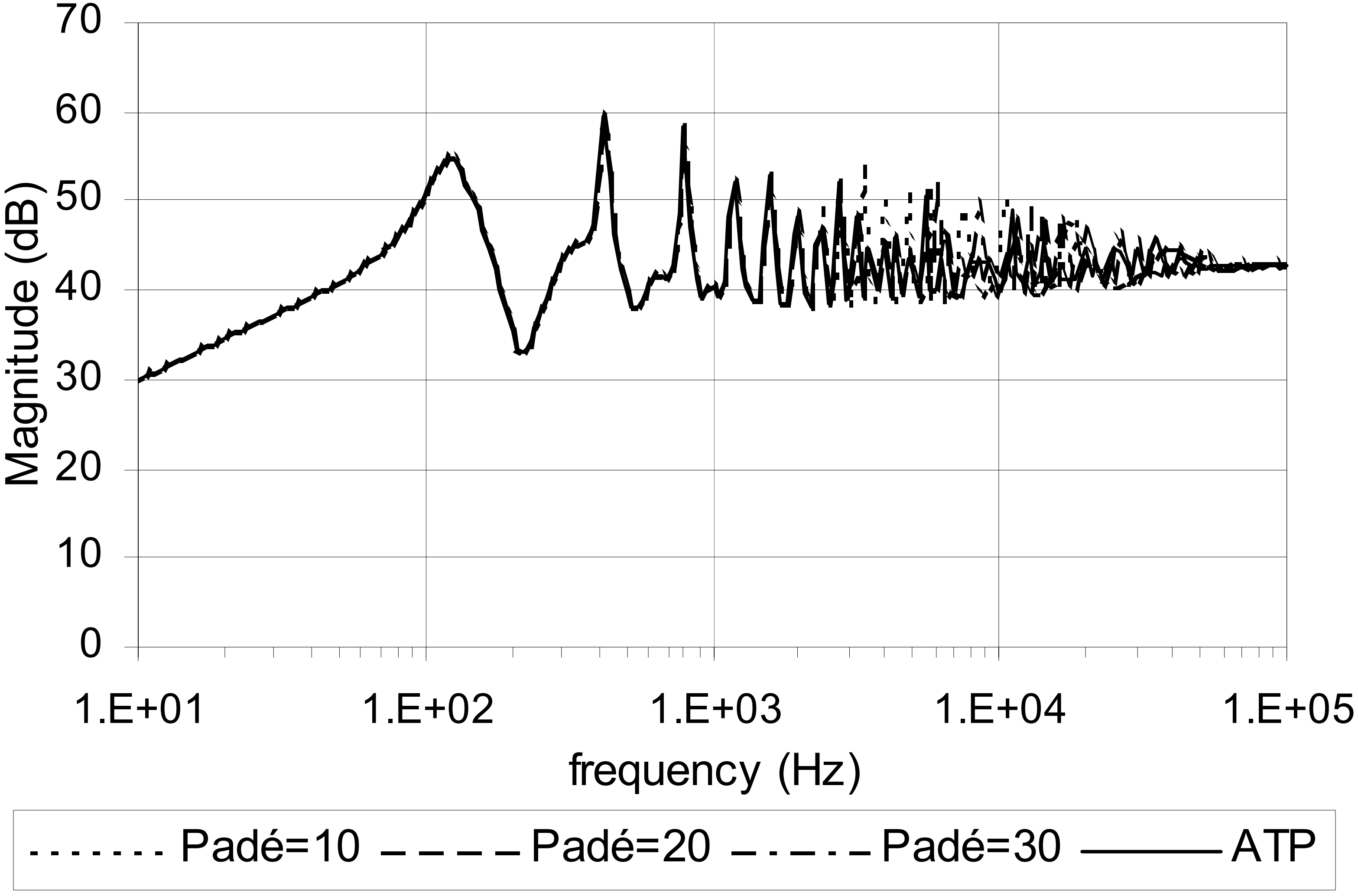
Test System



5
1
2
6
6,5 Ω
0,4 H
153 km
0,4 H
6,5 Ω
350 km
250 km
3
4
8
0,4 H
6,5 Ω
100 km
200 Ω
Z_eq
0,48 H

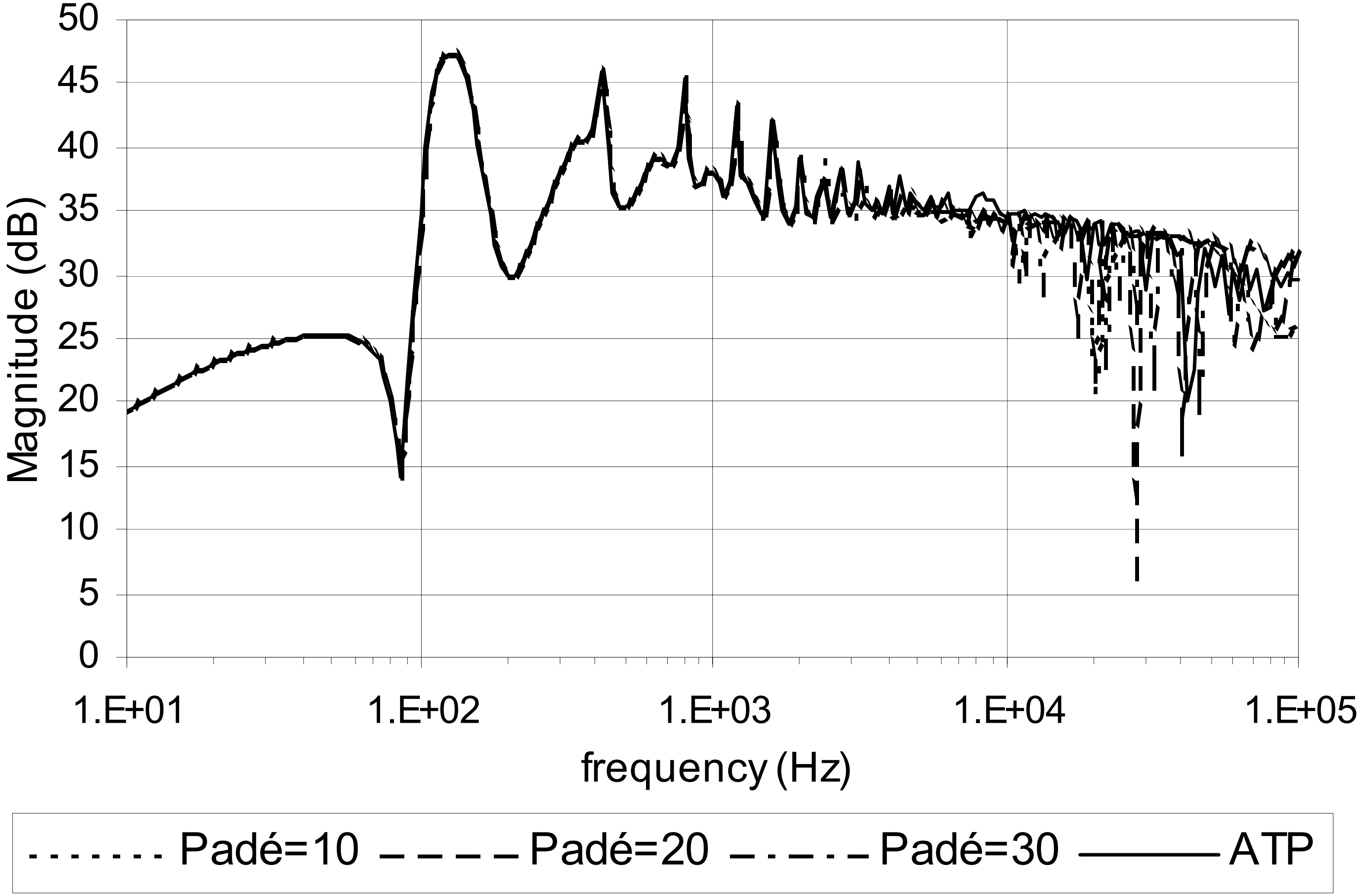
Results

$$\frac{V_{a3}(s)}{I_{a3}(s)}$$



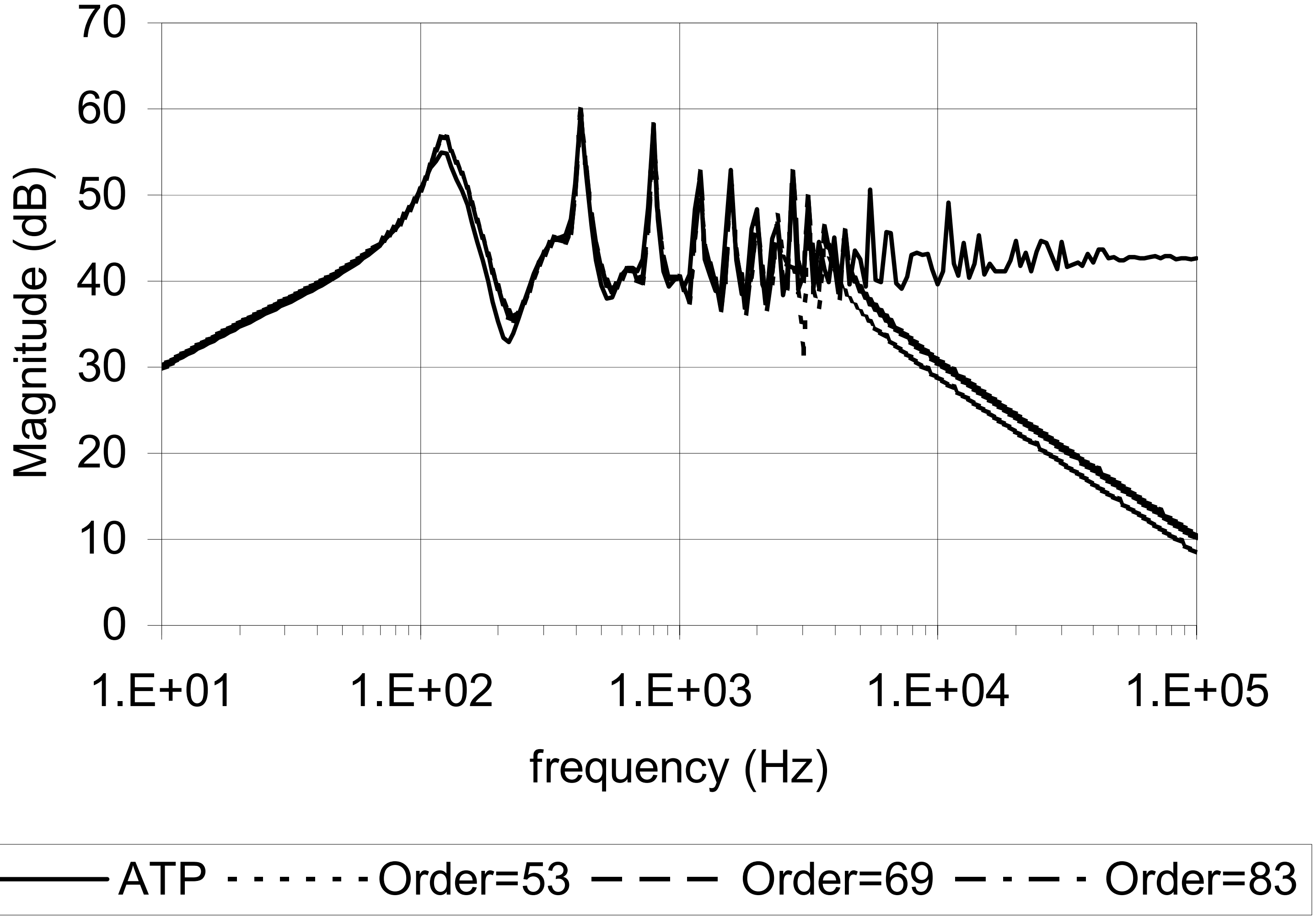
Results

$$\frac{V_{b3}(s)}{I_{a3}(s)}$$



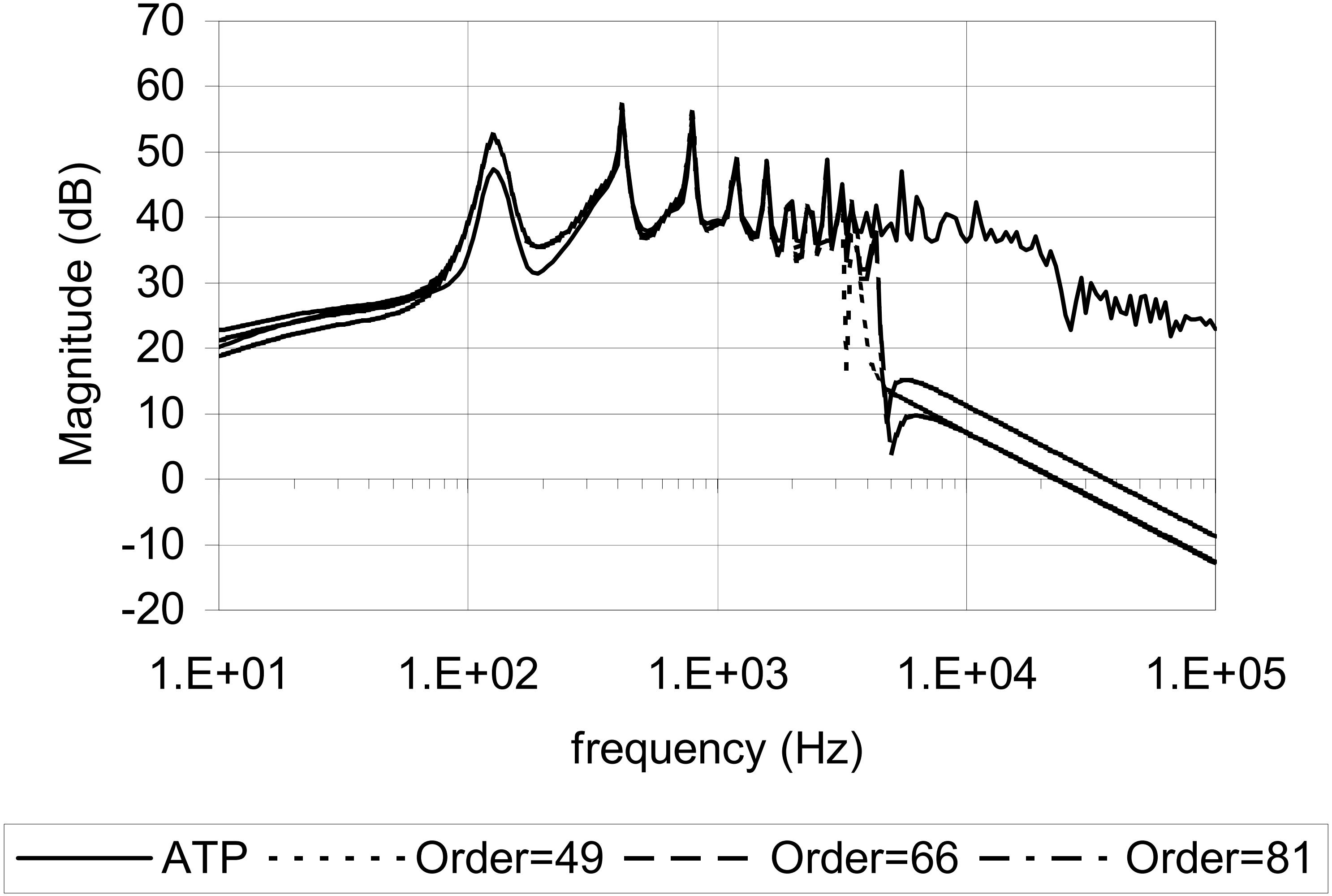
Reduced Order Equivalent

$$\frac{V_{a3}(s)}{I_{a3}(s)}$$



Reduced Order Equivalent

$$\frac{V_{c3}(s)}{I_{a3}(s)}$$



Conclusions

- State-space based formulations obtain the same frequency domain results as the conventional methods based on nodal formulation and more:
  - Modal analysis techniques can be used to obtain poles, zeros, transfer function residues, sensitivities, etc.
  - Reduced order dynamic equivalents obtained through use of transfer function residues

Conclusions

- Descriptor system approach allows:
  - Simple and efficient computational implementation
  - Ability to model systems of any topology and containing state variable redundancies
  - Applicability to large-scale networks, due to the very sparse matrices involved and the availability of powerful sparse eigensolution algorithms applied to descriptor systems