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Computing Dominant Poles of Power System Transfer Functions

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Transfer Function Pole Dominance

- Concept little exploited in Numerical Linear Algebra
- Power system applications: AESOPS algorithm [Byerly, 1978; Kundur, 1990] and Selective Modal Analysis [Pagola, 1988]
- Need for numerical efficiency, robustness and more general eigensolution selectivity in power system small signal stability analysis and decentralized control design

Dominant Pole Algorithm (DPA) (1/4)

1. Given a single-input-single-output transfer function,

$$G(s) = \mathbf{c}^{\mathbf{t}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}$$

Provide an initial eigenvalue estimate s_k and specify the value for the output at all iterations as unity ($y(s_k) = 1$), k being the iteration counter

Dominant Pole Algorithm (DPA) (2/4)

2. Solve

$$\begin{bmatrix} s_k \mathbf{I} - \mathbf{A} & -\mathbf{b} \\ \mathbf{c}^{\mathbf{t}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(s_k) \\ u(s_k) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$

3. Solve

$$\begin{bmatrix} s_k \mathbf{I} - \mathbf{A}^{t} & -\mathbf{c} \\ \mathbf{b}^{\mathbf{t}} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}(s_k) \\ u(s_k) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$

using the transpose of the LDU factors calculated at step 2

Dominant Pole Algorithm (DPA) (3/4)

4. Compute the new eigenvalue estimate s_{k+1}

$$s_{k+1} = s_k + \frac{u(s_k)}{\mathbf{v}^t(s_k) \cdot \mathbf{x}(s_k)}$$

This expression shows that s_{k+1} converges to the eigenvalue λ as the input $u(s_k)$ approaches zero.

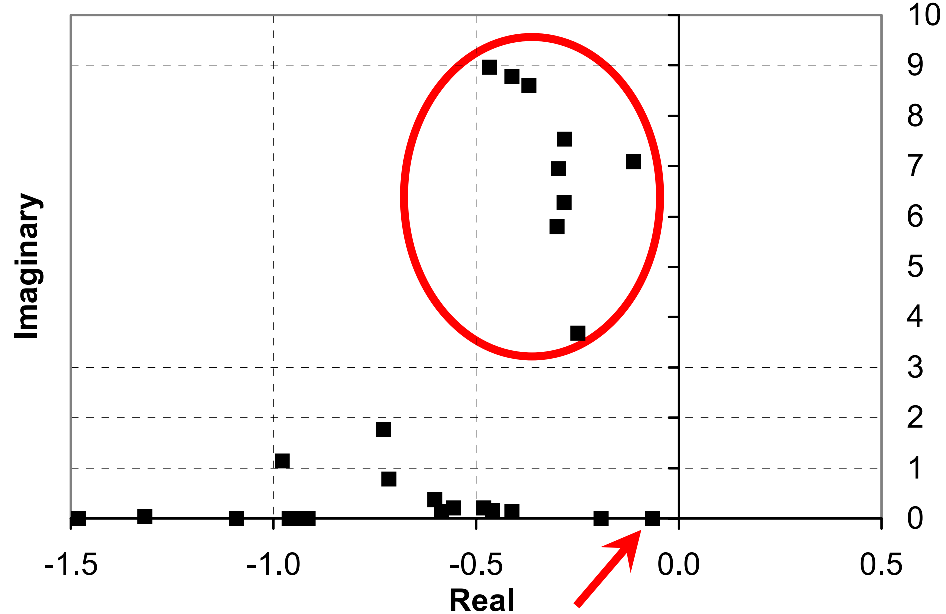
Dominant Pole Algorithm (DPA) (4/4)

5. If change in eigenvalue estimate ($\Delta s = s_k + 1 - s_k$) is greater than convergence tolerance increase the iteration counter ($k = k+1$) and return to step 2. Otherwise the algorithm has converged to an eigenvalue which is a dominant pole in $G(s)$, and to its left and right eigenvectors.

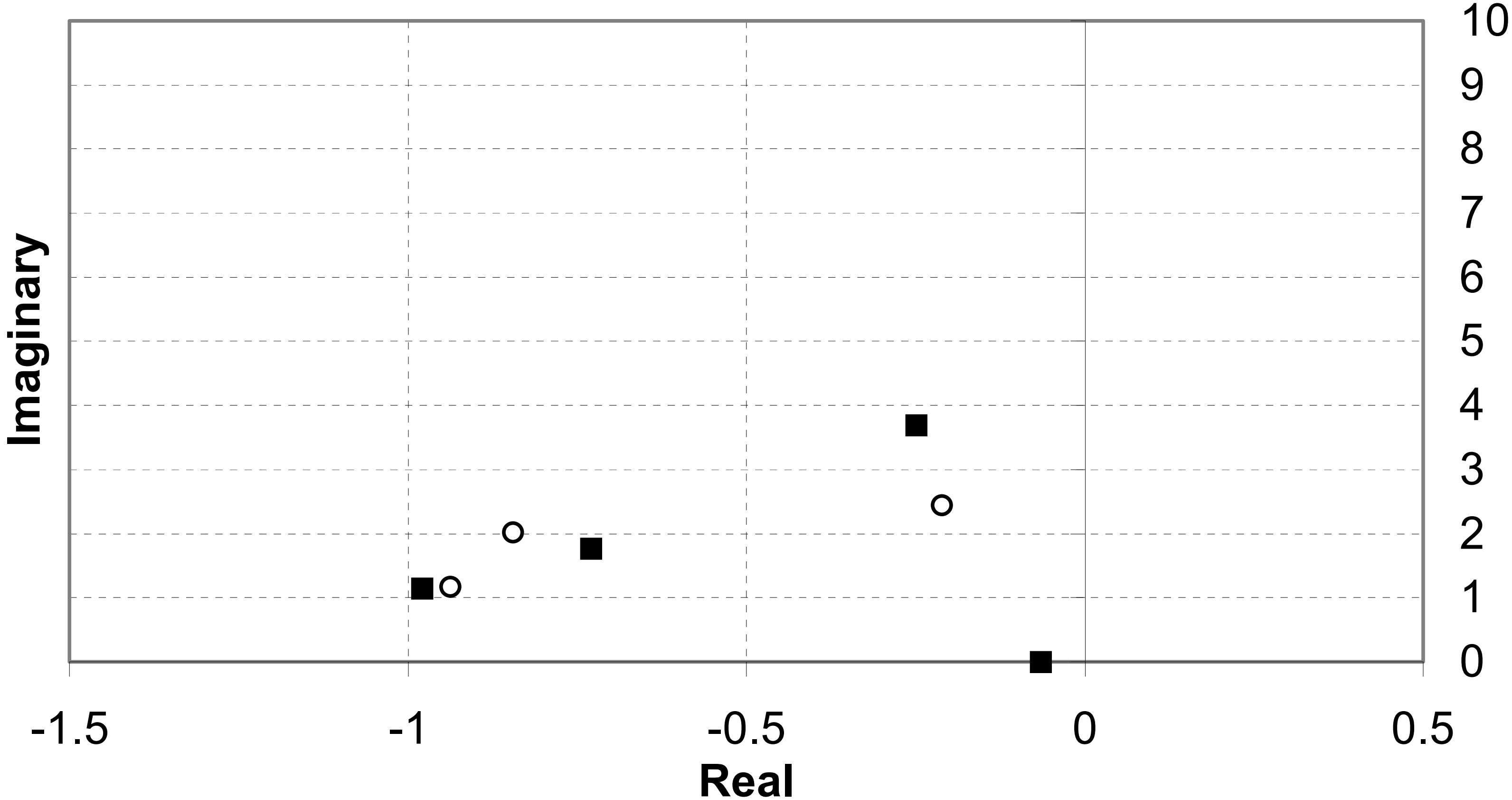
Note:
The practical implementation of this algorithm to the power system stability model operates on the sparse non-reduced Jacobian, but for the sake of clarity it is here described as operating directly on the state matrix **A**.

New England System Eigenvalues

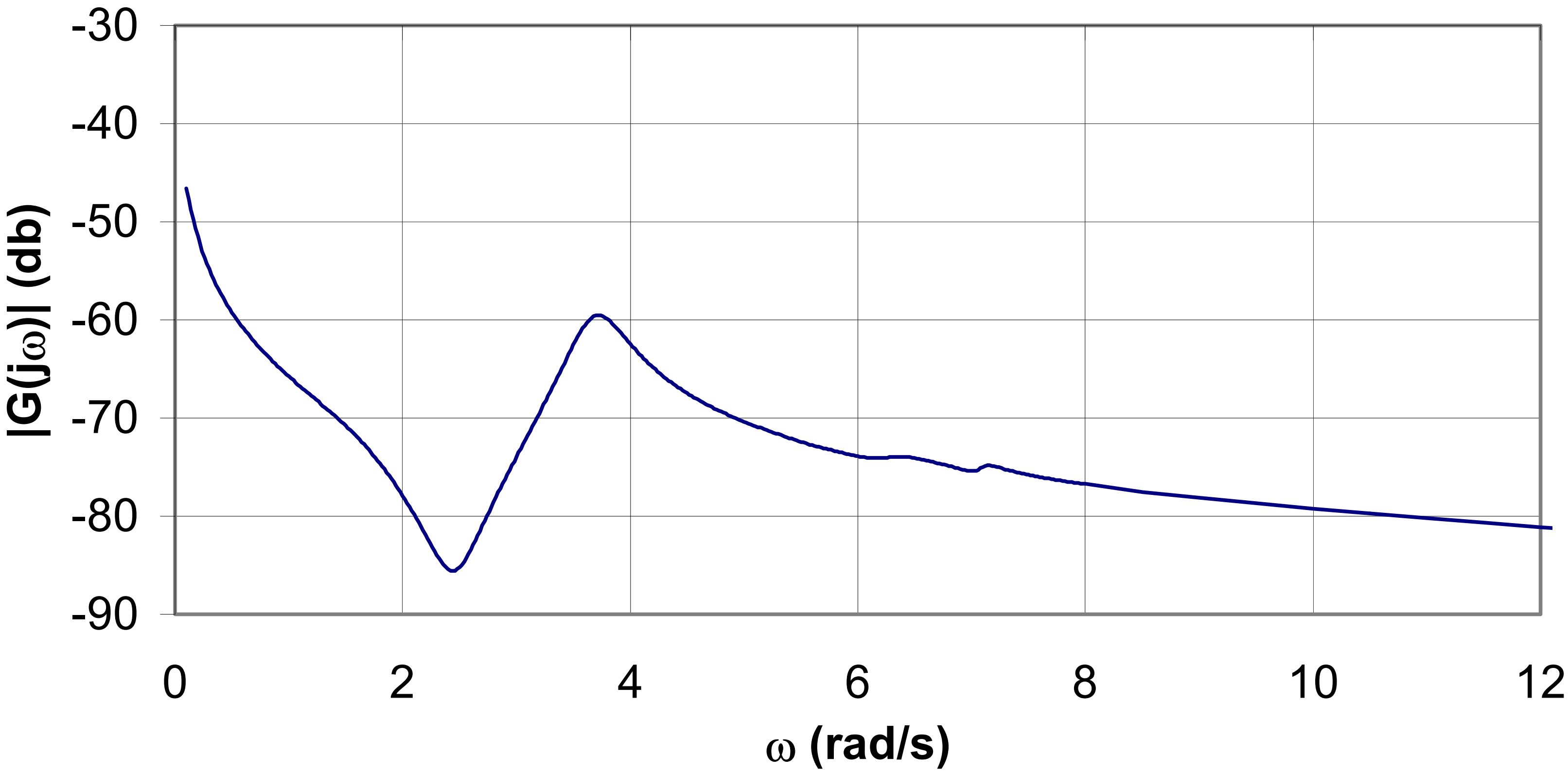
	Eigenvalue	Generators with Highest Participation
1	$-0.467 \pm j8.965$	36, 35
2	$-0.412 \pm j\ 8.779$	37
3	$-0.370 \pm j\ 8.611$	33
4	$-0.282 \pm j\ 7.537$	32, 31
5	$-0.112 \pm j\ 7.095$	30
6	$-0.297 \pm j\ 6.956$	35, 36, 31
7	$-0.283 \pm j\ 6.282$	31, 32, 34, 38
8	$-0.301 \pm j\ 5.792$	38, 34
9	$-0.249 \pm j\ 3.686$	39, 38, 34



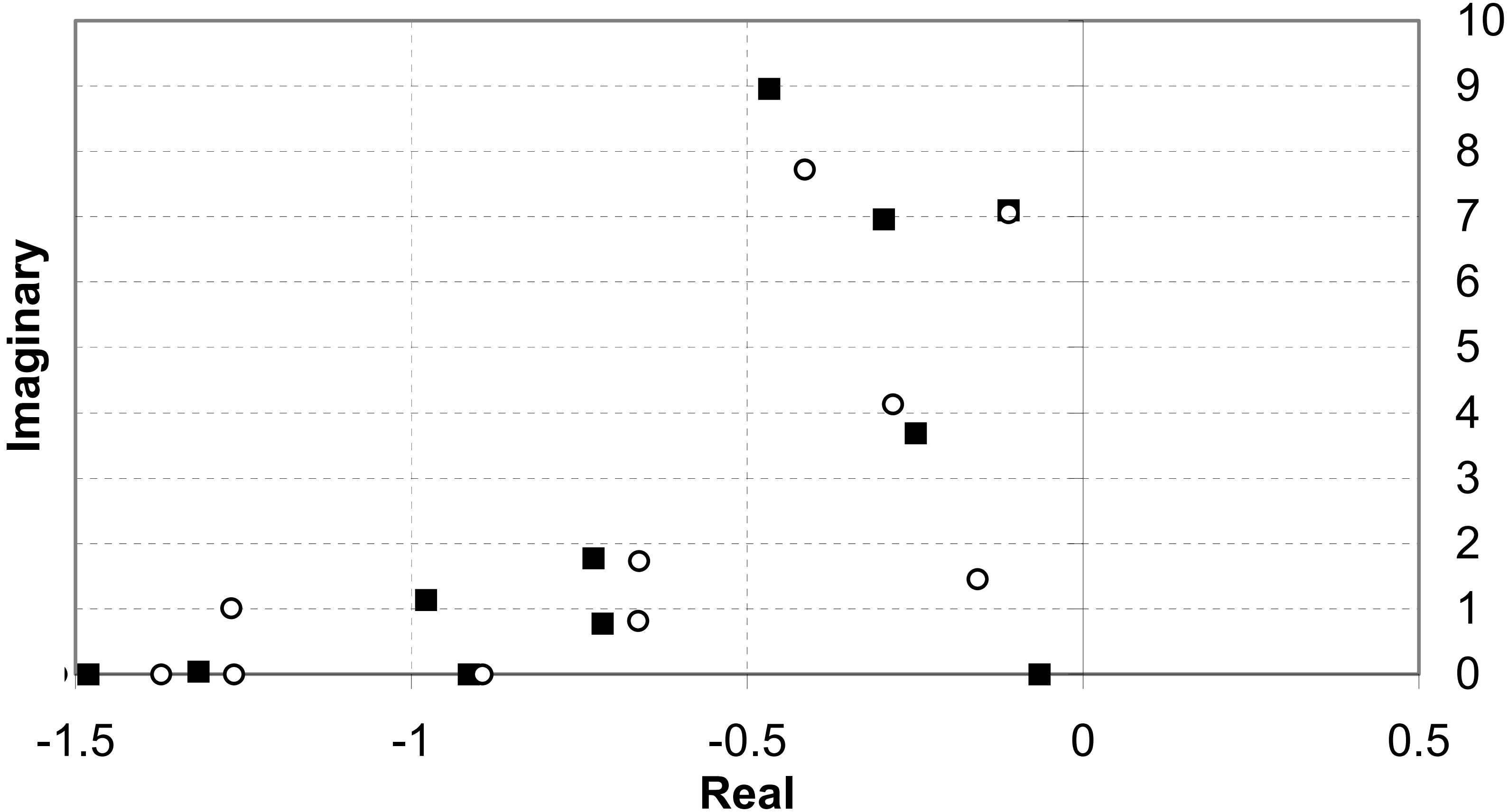
Dominant Pole-Zero Spectrum Plot for $\Delta\omega^{39}(s)/\Delta P_{mec}^{39}(s)$



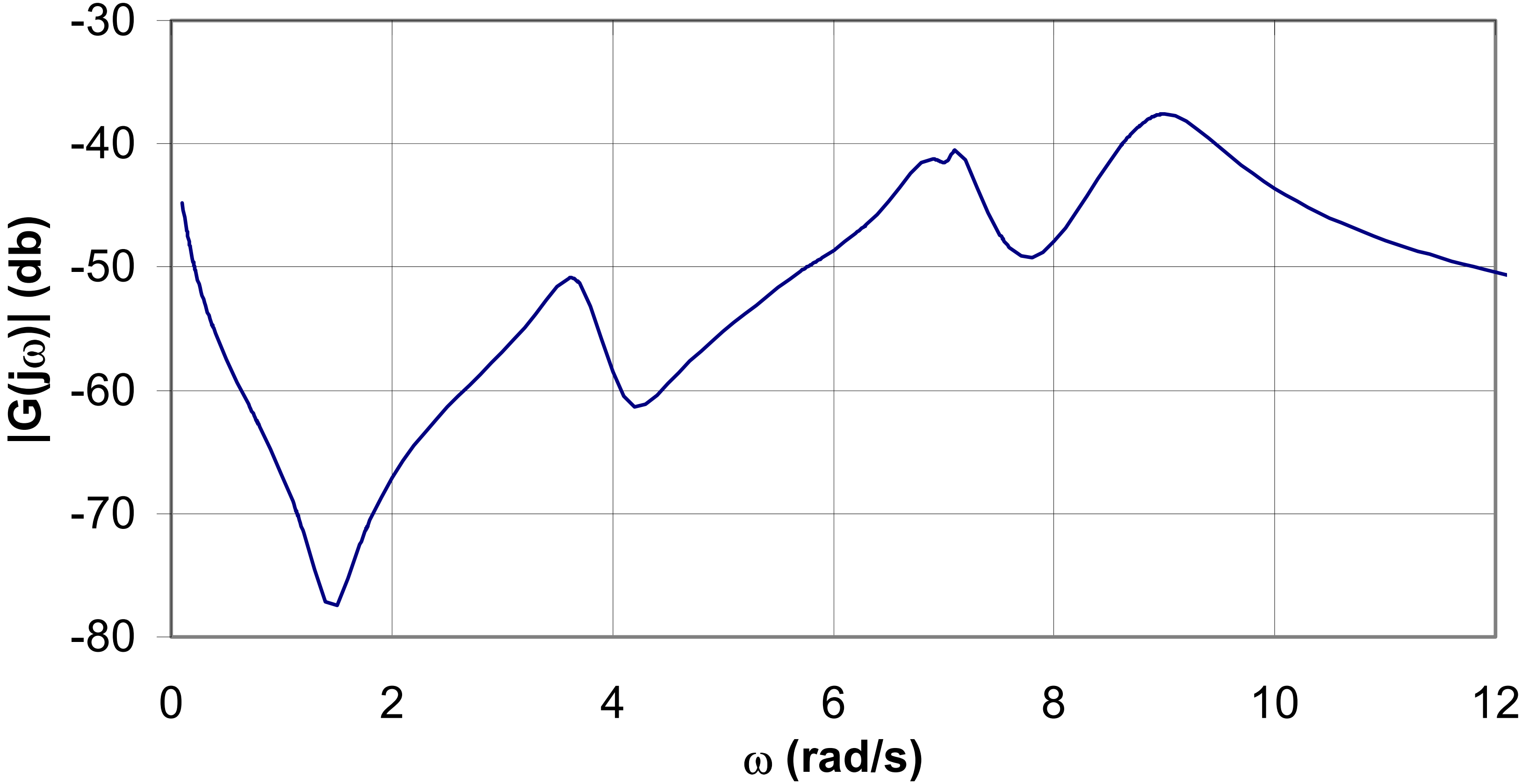
Bode Magnitude Plot for $\Delta\omega^{39}(s)/\Delta P_{mec}^{39}(s)$



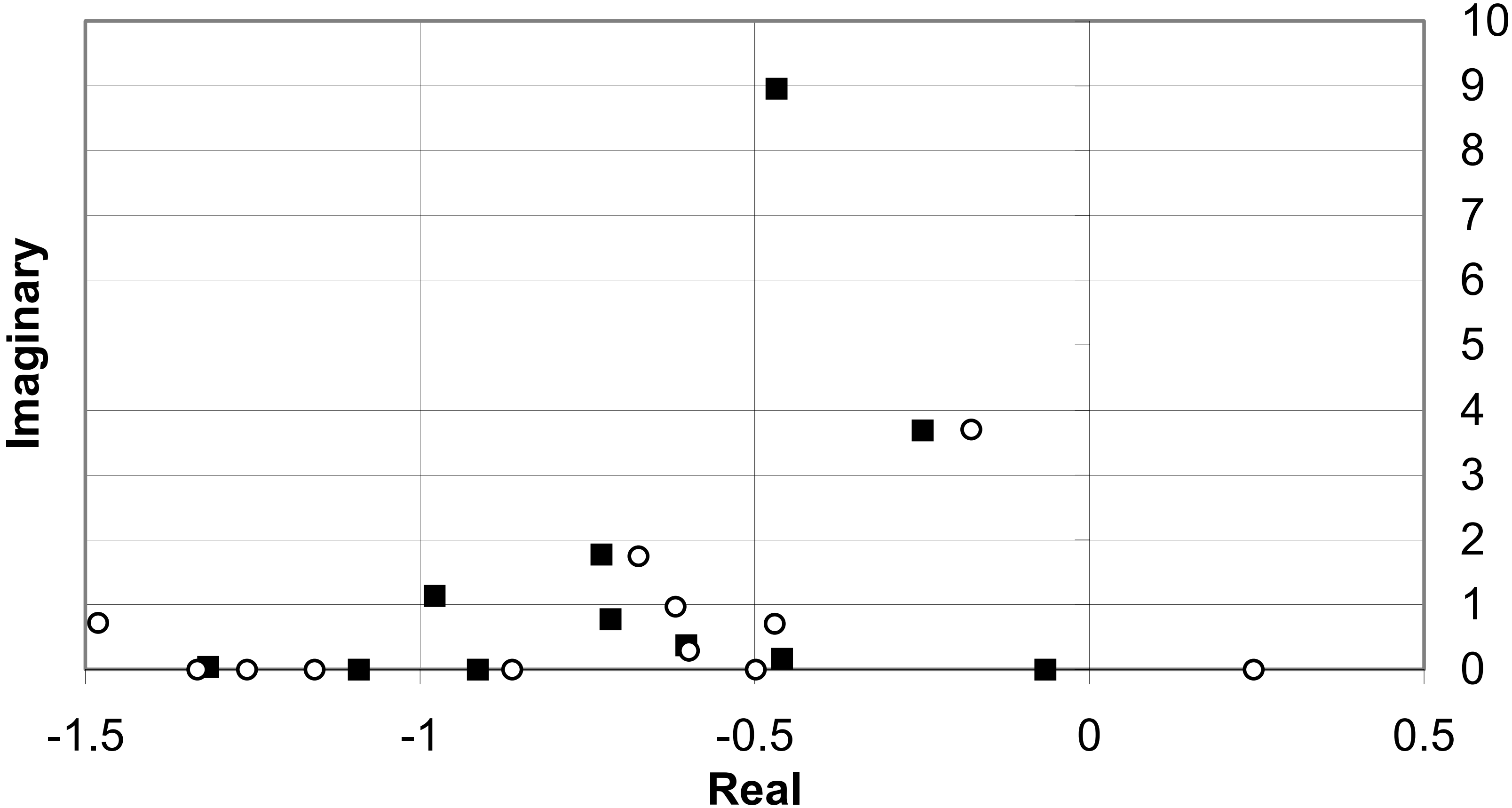
Dominant Pole-Zero Spectrum Plot for $\Delta\omega^{36}(s)/\Delta P_{mec}{}^{36}(s)$



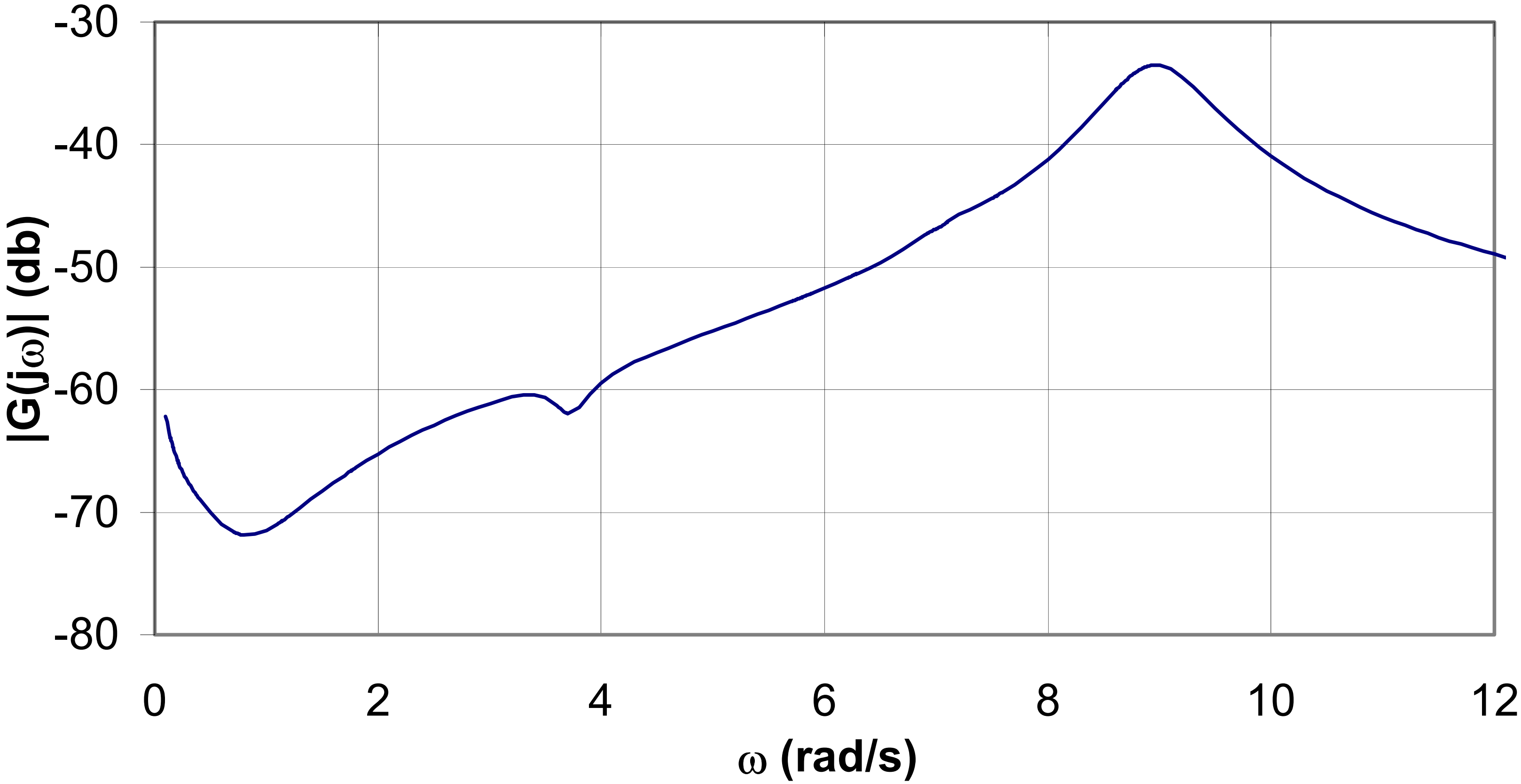
Bode Magnitude Plot for $\Delta\omega^{36}(s)/\Delta P_{mec}^{36}(s)$



Dominant Pole-Zero Spectrum Plot for $\Delta\omega^{36}(s)/\Delta P_{mec}^{35\text{-}36}(s)$



Bode Magnitude Plot for $\Delta\omega^{36}(s)/\Delta P_{mec}^{35\text{-}36}(s)$



Transfer Function Dominant Poles Obtained for Widely Different Initial Estimates

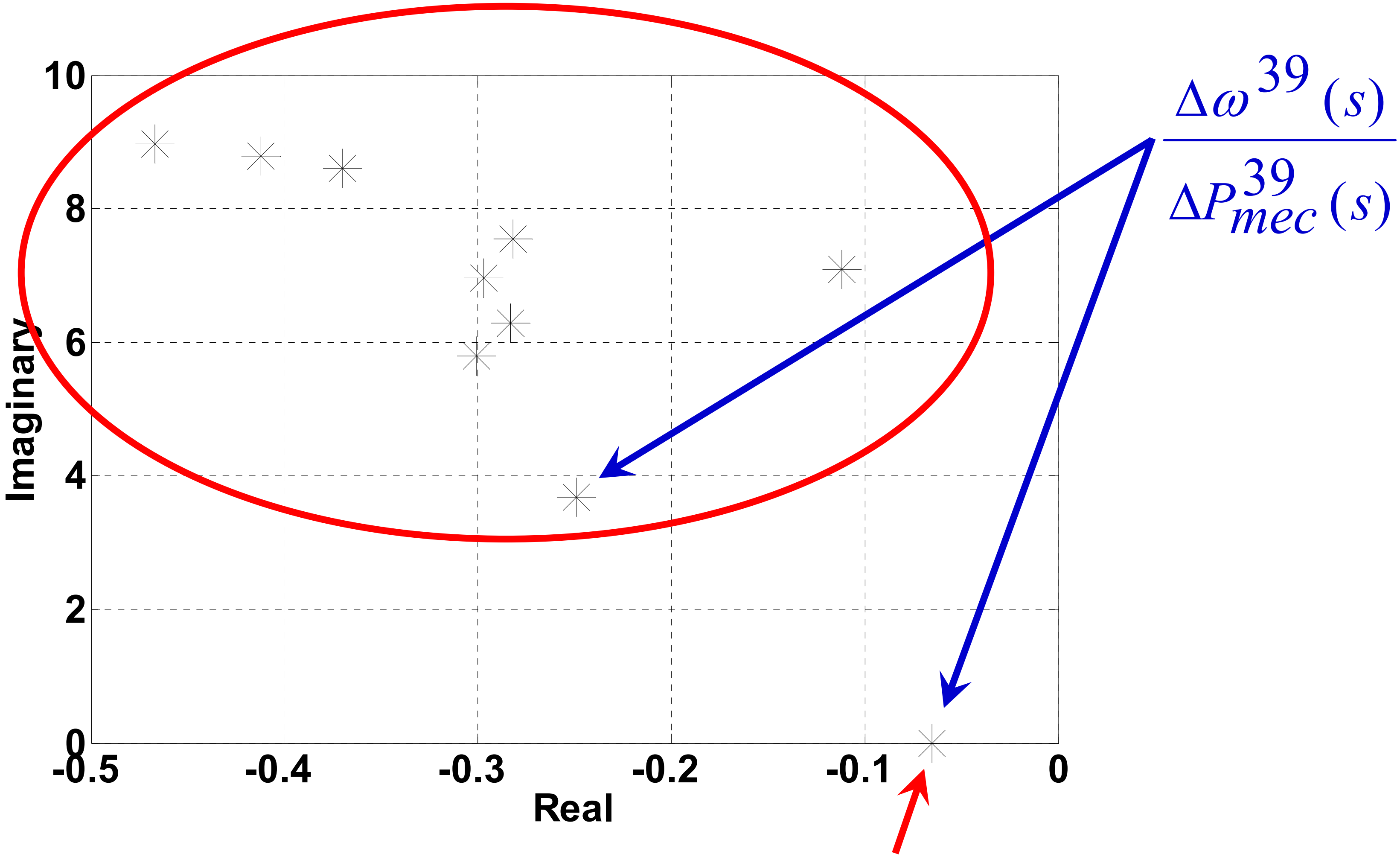
Initial Estimate s_k	Transfer Functions		
	$\frac{\Delta\omega^{39}(s)}{\Delta P_{mec}^{39}(s)}$	$\frac{\Delta\omega^{36}(s)}{\Delta P_{mec}^{36}(s)}$	$\frac{\Delta\omega^{36}(s)}{\Delta P_{mec}^{35-36}(s)}$
$j3$	λ_1 (5)	λ_1 (6)	λ_3 (5)
$j5$	λ_1 (6)	λ_2 (7)	λ_3 (6)
$j7$	λ_1 (8)	λ_2 (7)	λ_3 (4)
$j9$	λ_1 (5)	λ_3 (4)	λ_3 (2)
$j200$	λ_1 (10)	λ_3 (9)	λ_3 (7)
$j10^{10}$	λ_4 (14)	λ_3 (12)	λ_3 (11)

$\lambda_1 = -0.249 \pm j3.686 \quad \lambda_2 = -0.297 \pm j6.956$

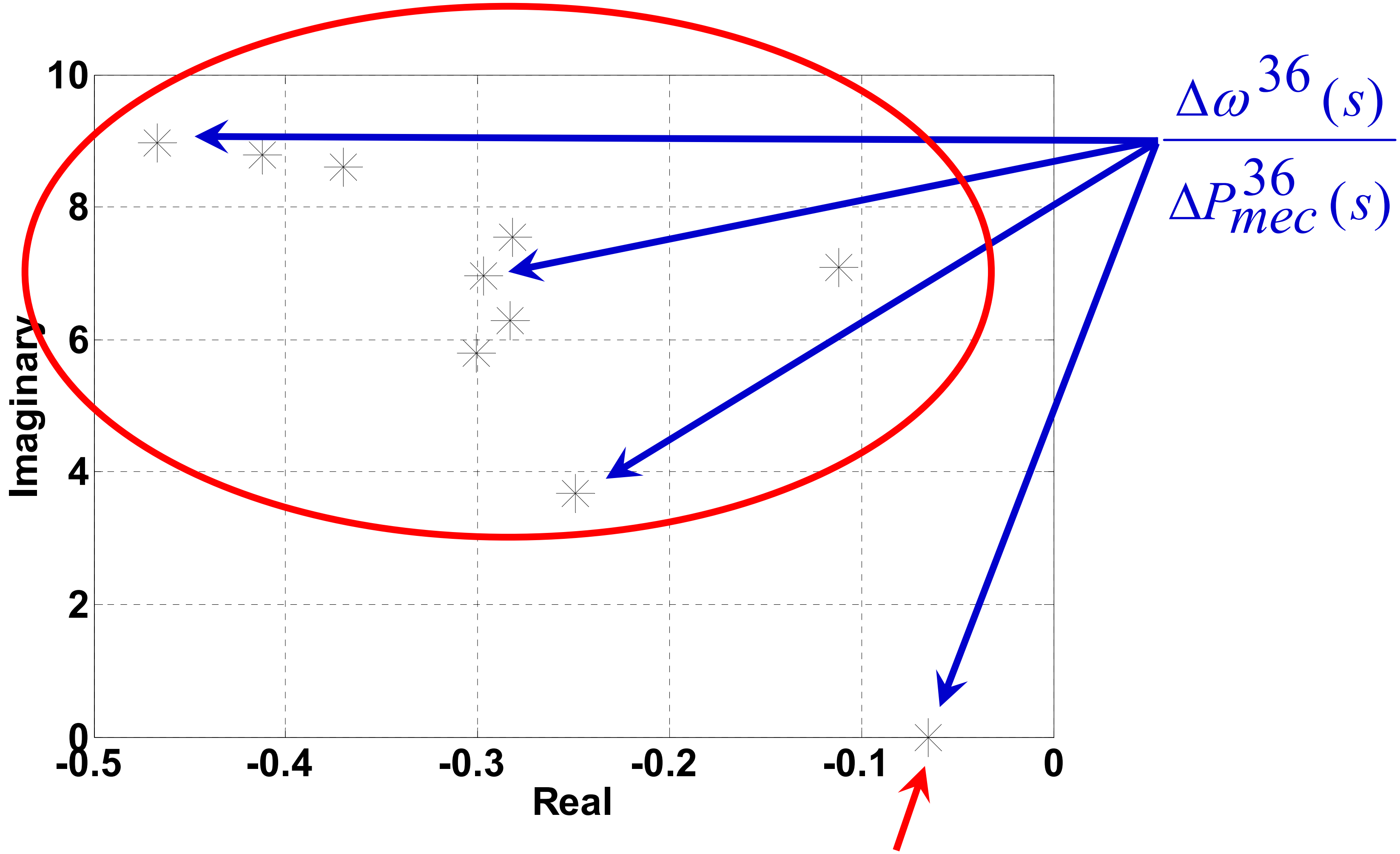
$\lambda_3 = -0.467 \pm j8.965 \quad \lambda_4 = -0.065$

Note: The numbers within parenthesis denote iterations required for convergence tolerance of 10^{-10}

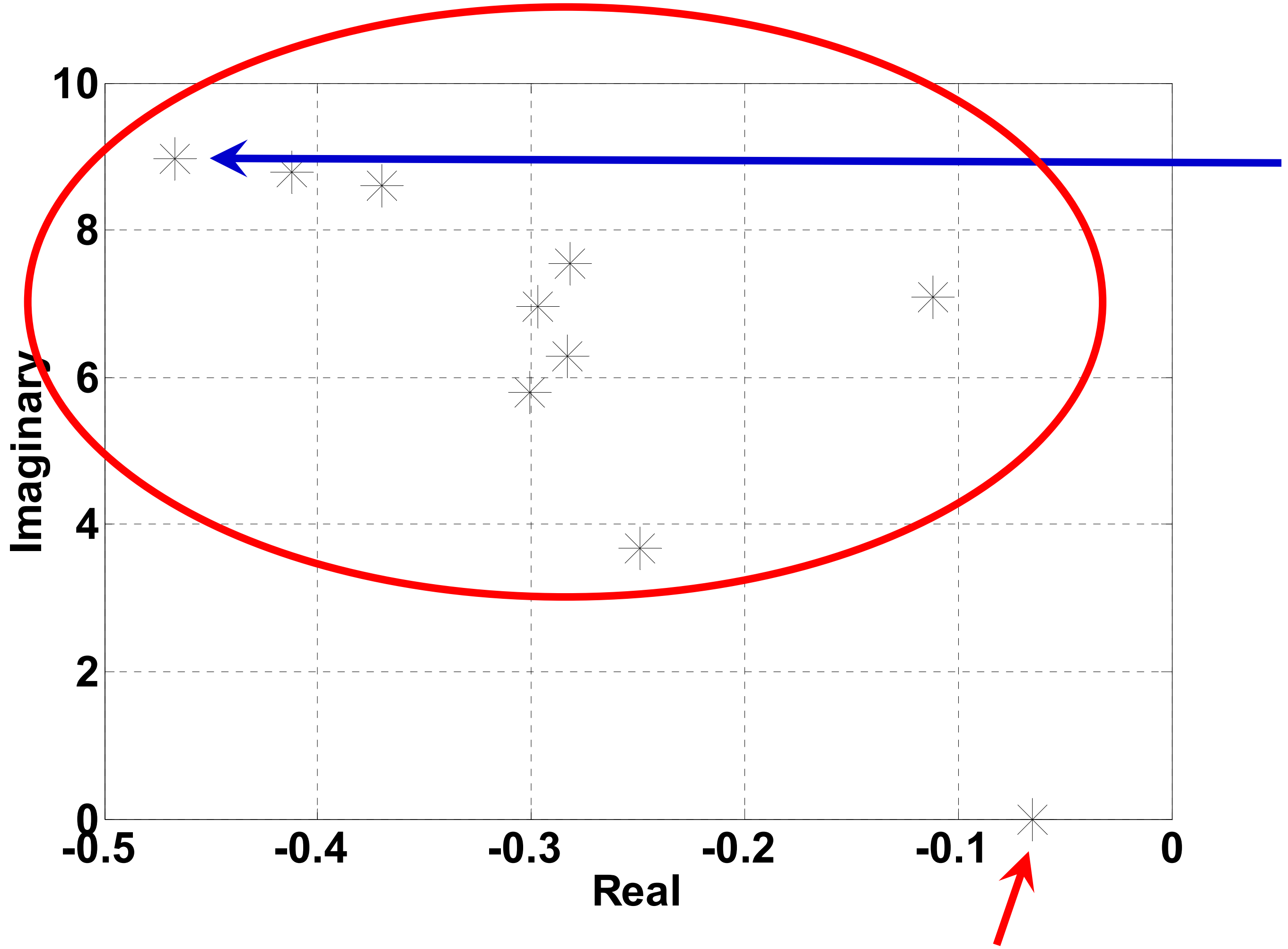
DPA Obtaining Selective Eigensolutions



DPA Obtaining Less Selective Eigensolutions

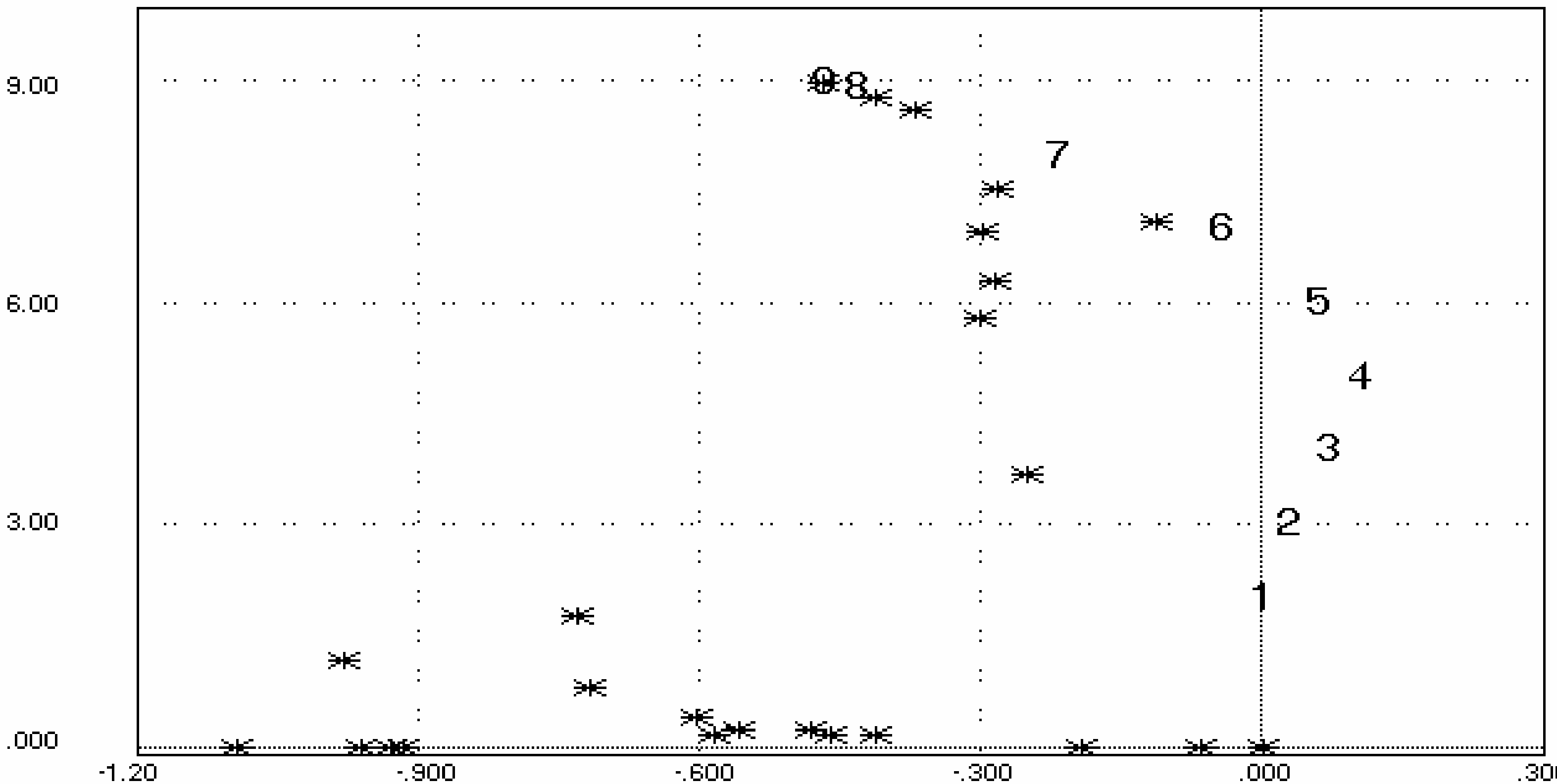


DPA Obtaining Very Selective Eigensolutions



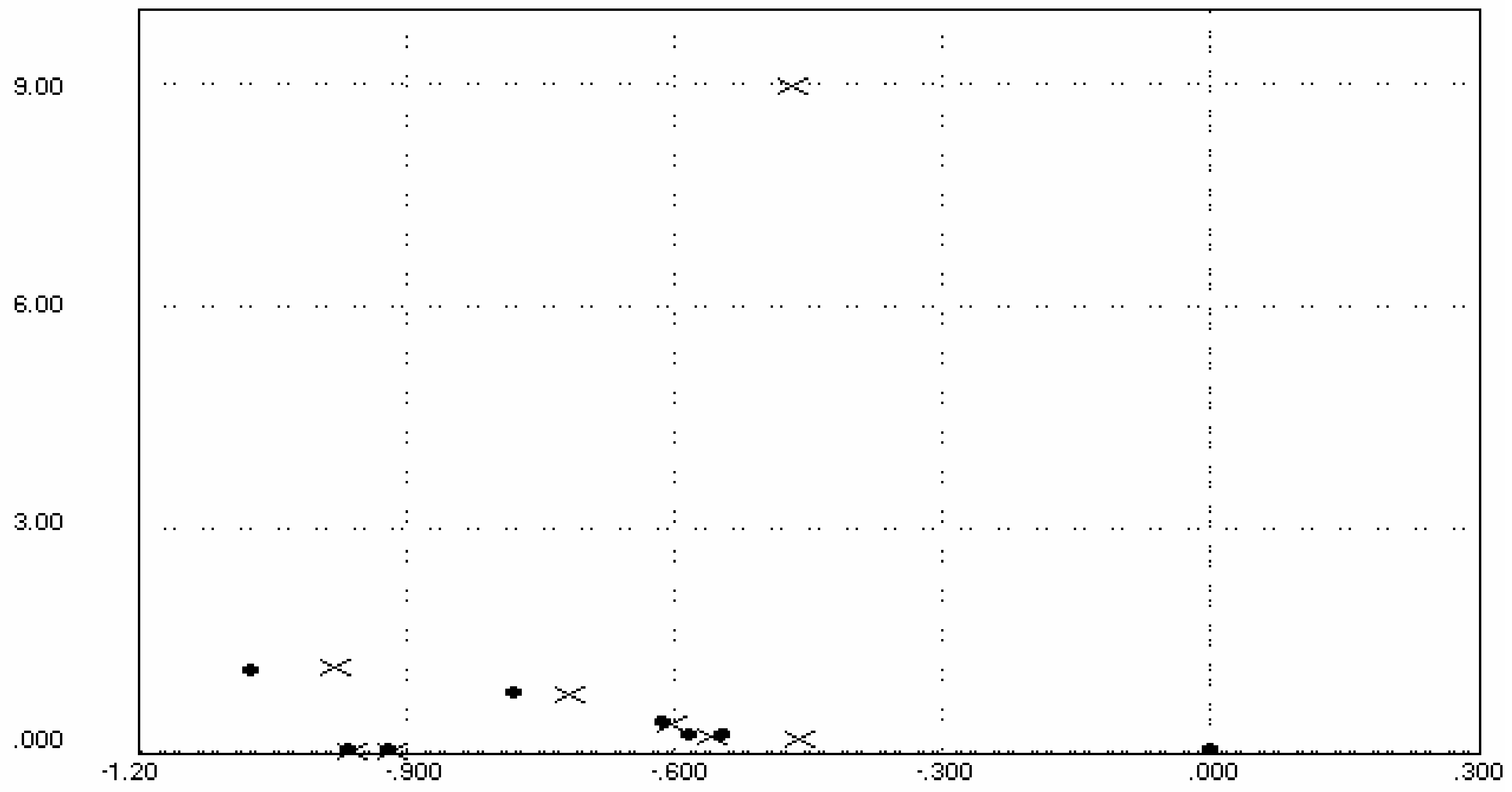
$$\frac{\Delta\omega^{36}(s)}{\Delta P_{mec}^{35-36}(s)}$$

Selective Convergence of Dom. Pole Algorithm



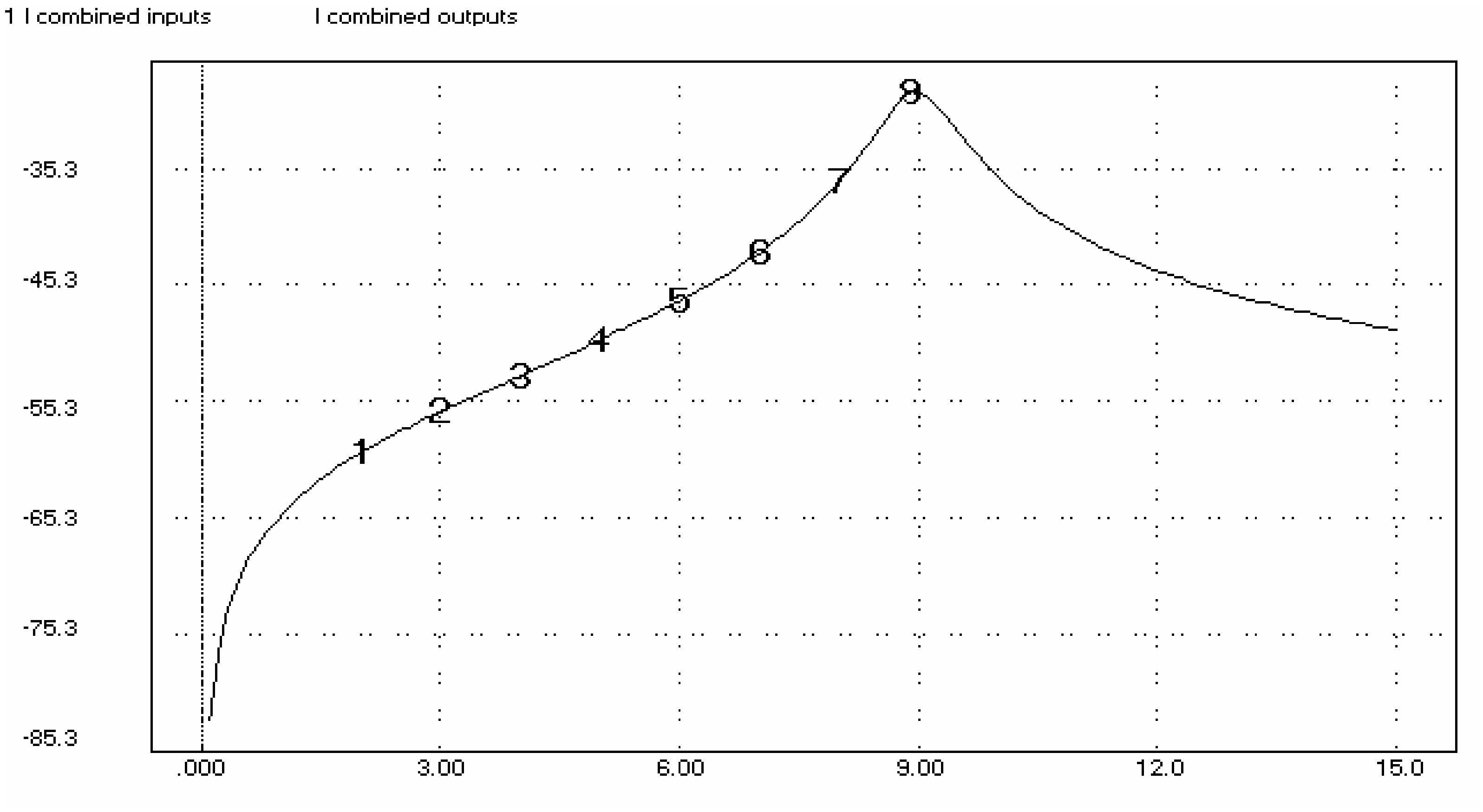
Trajectory of eigenvalue estimate with final convergence to the dominant pole of transfer function $\Delta\omega^{35\text{-}36}(s)/\Delta P_{mec}^{35\text{-}36}(s)$ of the New England system

Pole-Zero Spectrum Seen by Dominant Pole Algorithm (DPA)



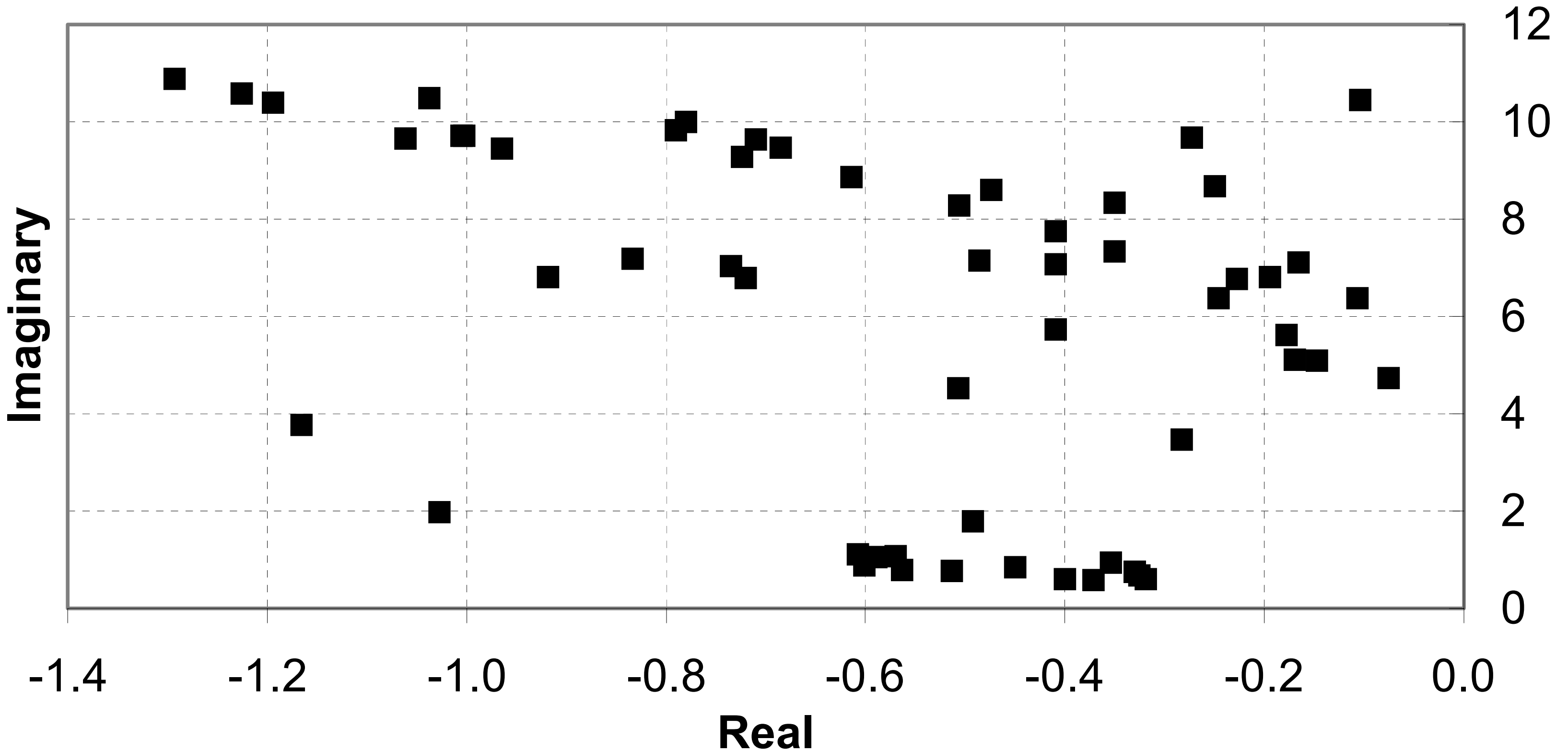
Dominant pole-zero spectrum for transfer function $\Delta\omega^{35\text{-}36}(s) \,/\, \Delta P_{mec}{}^{35\text{-}36}(s)$ of the New England system

Trajectory of Eigenvalue Estimate for DPA

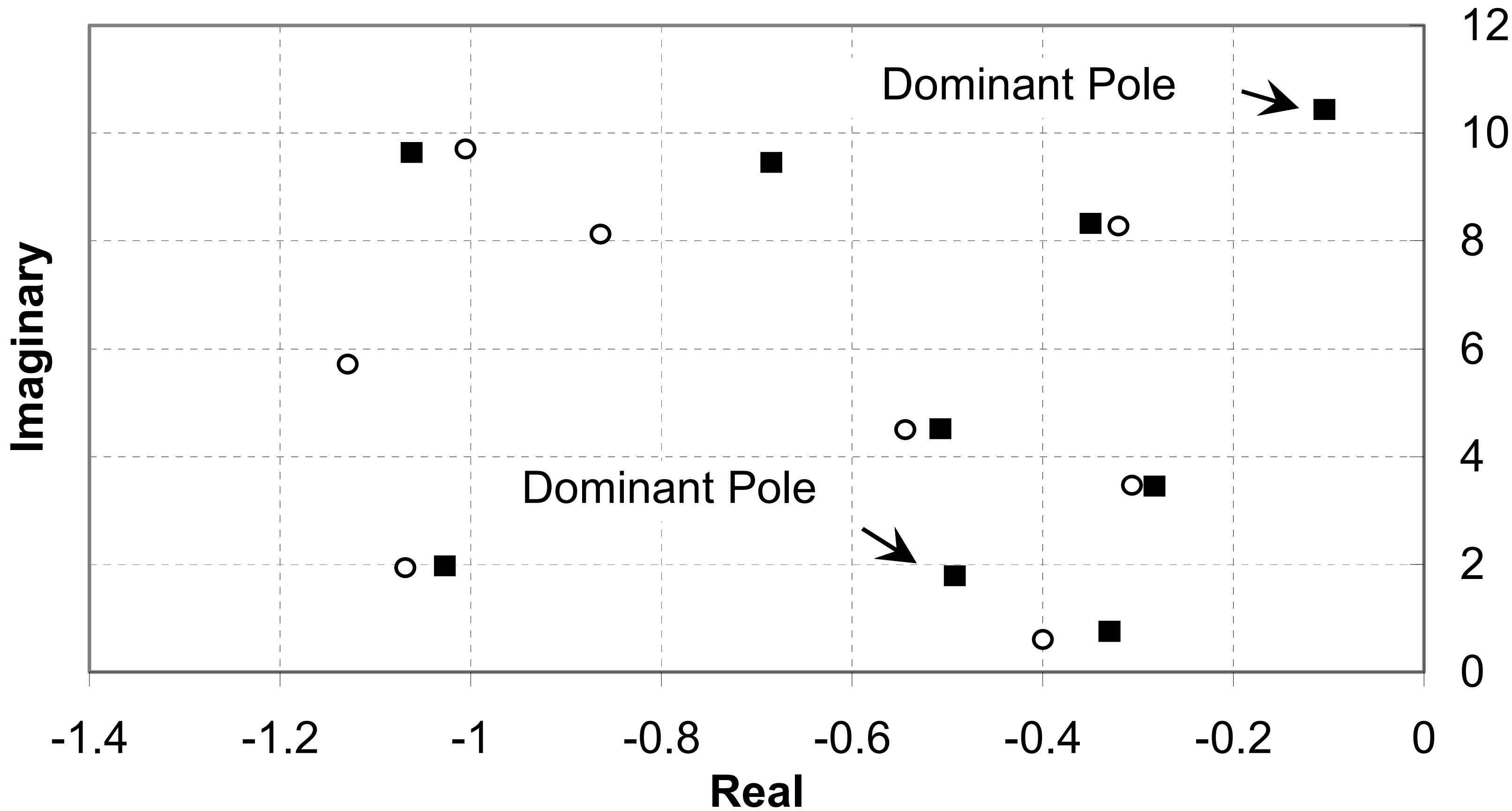


Bode magnitude plot for transfer function $\Delta\omega^{35\text{-}36}(s)/\Delta P_{mec}{}^{35\text{-}36}(s)$ of the New England test system

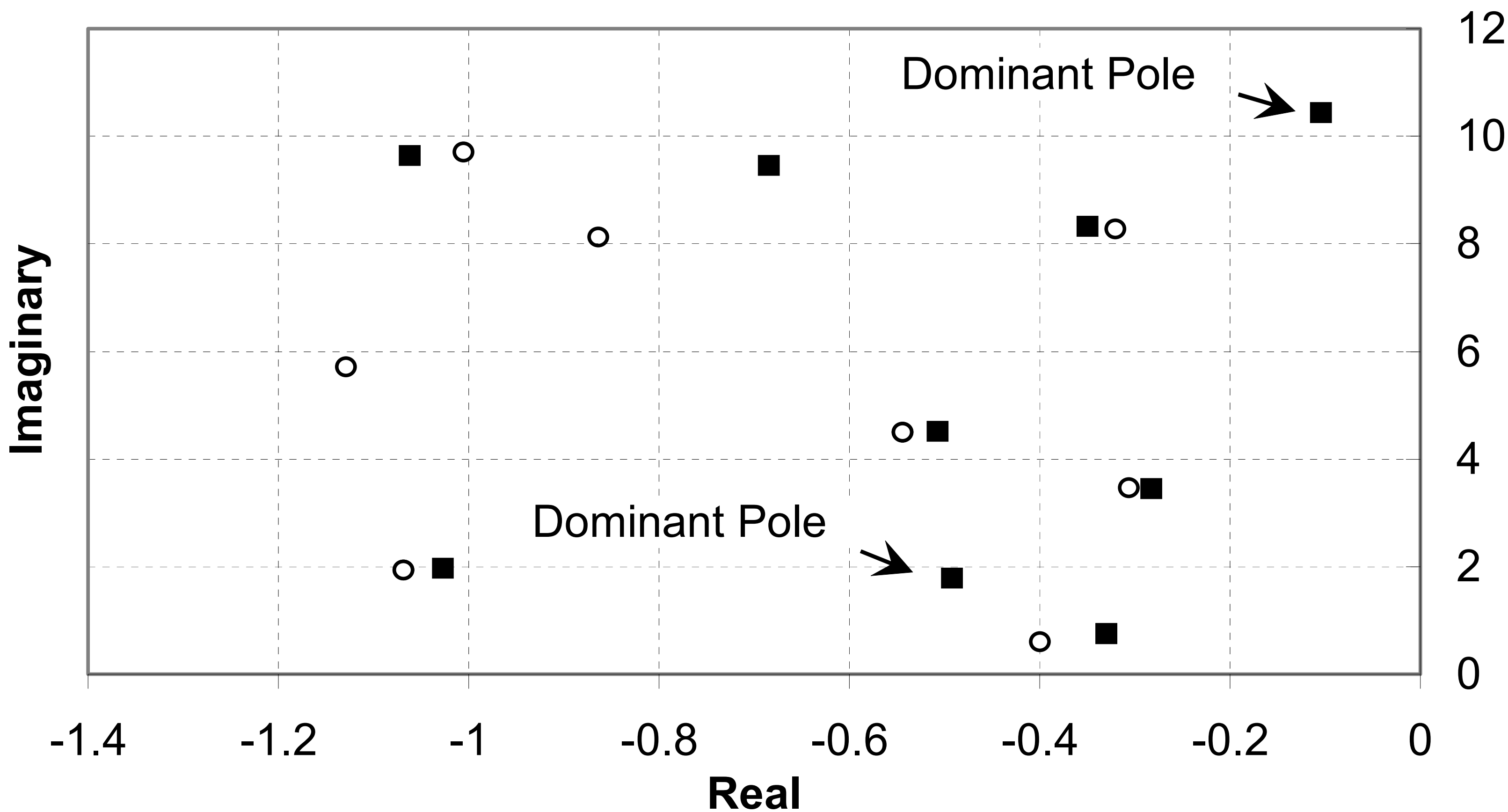
Large System Results



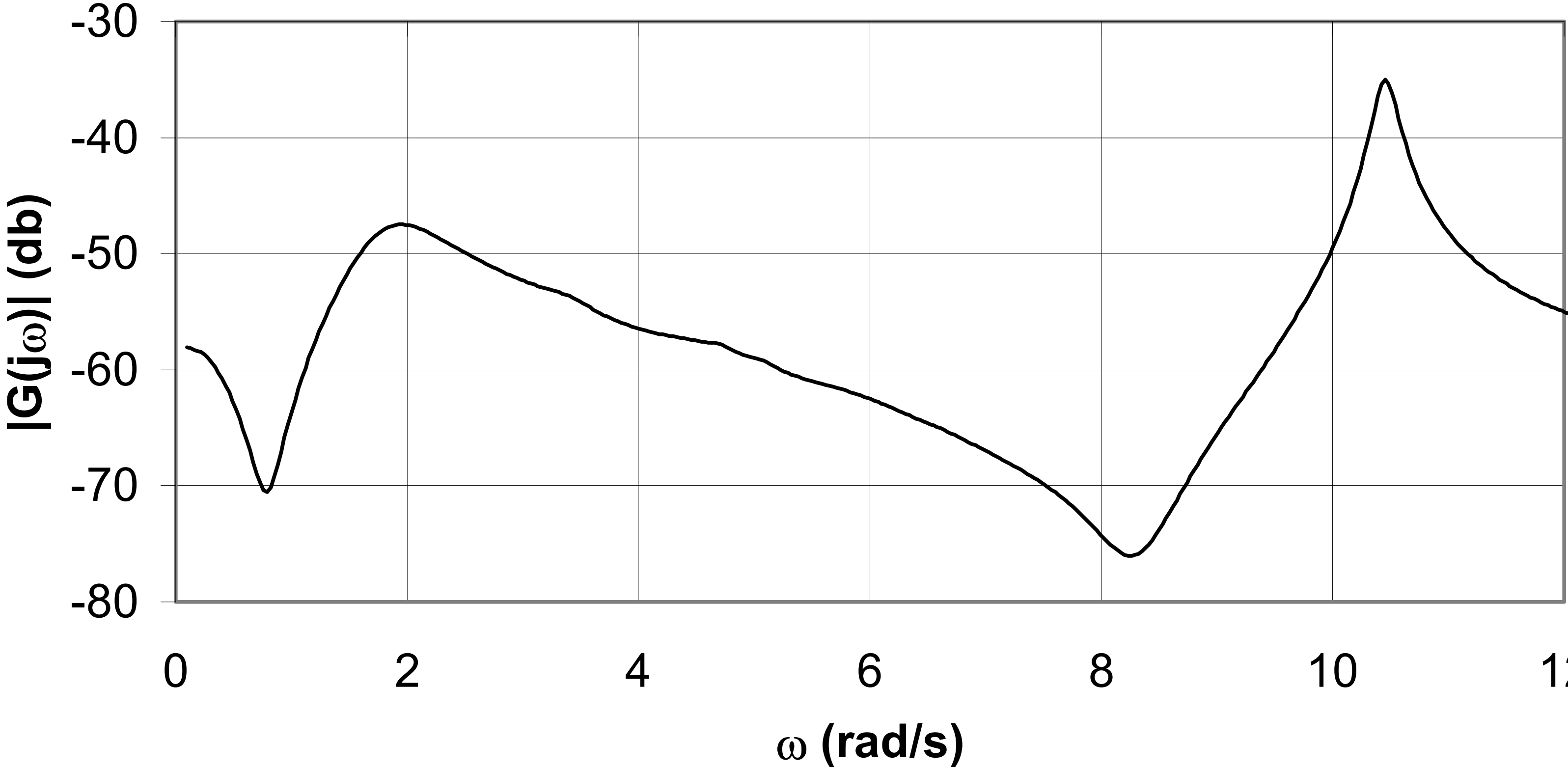
Eigenvalues of the Large Power System Stability Model



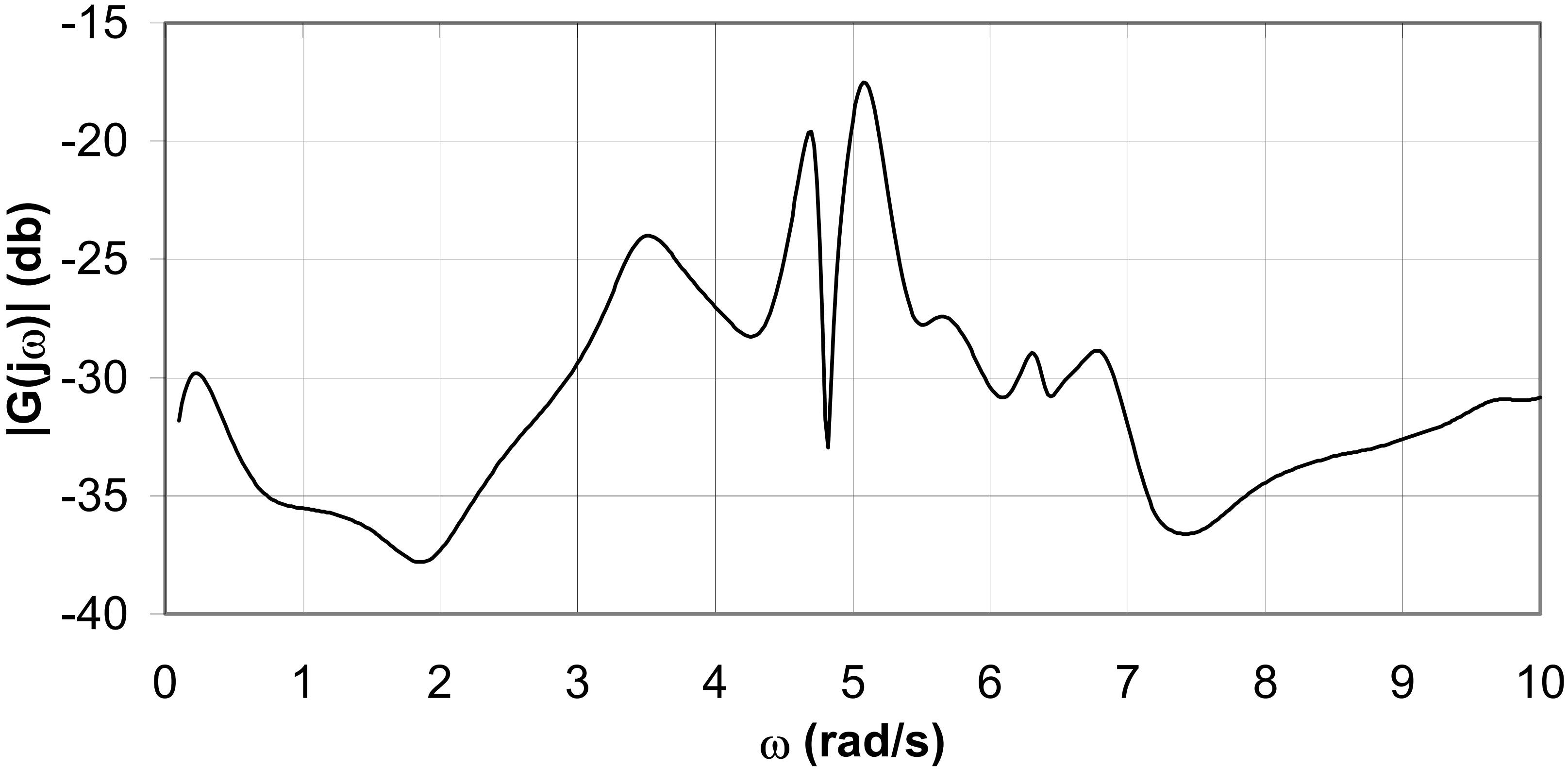
Dominant Pole-Zero Spectrum Plot for $\Delta\omega(s)/\Delta P_{mec}(s)$ for Generator #303



Bode Magnitude Plot for $\Delta\omega(s)/\Delta P_{mec}(s)$ for Generator #303



Bode Magnitude Plot for $\Delta Q_{flow}(s)/\Delta I_{order}(s)$



Transfer Function Dominant Eigenvalues Obtained for Several Initial Estimates

Initial Estimate s_k	Transfer Functions	
	$\Delta\omega(s)/\Delta P_{mec}(s)$	$\Delta Q_{flow}(s)/\ \Delta I_{order}(s)$
$j1$	$-0.493 + j\ 1.793$ (7)	$-0.583 + j\ 0.585$ (8)
$j2$	$-0.493 + j\ 1.793$ (6)	$-0.283 + j\ 3.462$ (10)
$j3$	$-0.493 + j\ 1.793$ (7)	$-0.507 + j\ 4.527$ (18)
$j4$	$-0.493 + j\ 1.793$ (7)	$-0.283 + j\ 3.462$ (15)
$j5$	$-0.493 + j\ 1.793$ (9)	$-0.147 + j\ 5.098$ (5)
$j6$	$-0.493 + j\ 1.793$ (10)	$-2.028 + j\ 5.085$ (14)
$j7$	$-0.493 + j\ 1.793$ (10)	$-0.816 + j\ 7.741$ (10)
$j8$	$-0.493 + j\ 1.793$ (15)	$-4.813 + j\ 7.575$ (14)
$j9$	$-0.104 + j\ 10.449$ (7)	$-4.150 + j\ 9.160$ (23)
$j10$	$-0.104 + j\ 10.449$ (5)	$-4.813 + j\ 7.575$ (15)
$j2000$	$-1.258 + j\ 0.002$ (17)	$-7.261 + j\ 7.308$ (17)

Note: The numbers within parenthesis denote iterations required for convergence tolerance of 10^{-10}

Conclusions

- The Dominant Pole Algorithm (DPA) may be applied to linearized state space models of large dynamic systems
- DPA is a sparse matrix implementation of a Newton-Raphson algorithm that solves for the poles of scalar transfer functions *F(s)*
- Convergence domain of a given eigensolution becomes larger for poles with high controllability/observability in *F(s)*
- Subdominant poles of *F(s)* may be efficiently obtained by sparse eigenvalue deflation techniques