DECOMPOSITION TECHNIQUE FOR EFFICIENT COMPUTATION OF SMALL-SIGNAL STABILITY PROBLEMS IN LARGE POWER SYSTEMS

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Abstract - This paper describes a decomposition technique to be used in connection with algorithms for the solution of small-signal stability problems of large power systems [1]. This technique decomposes the complete system into one internal system and several interconnected external systems and allows efficient use of parallel computation. This technique is also valuable in frequency response studies, on a uniprocessor computer, when the applied disturbance and monitored outputs are all located within the internal system.

I. INTRODUCTION

Reference 1 presents efficient algorithms for the solution of small-signal stability problems of large power systems. These problems include the calculation of dominant eigenvalues, frequency response plots of transfer functions between any two variables in the system and step response solutions by numerical integration.

This paper describes a technique to be used in connection with these algorithms which decomposes the complete electric system into one internal system and several external systems. The formulation developed is general and each external system can be connected to any other external system and to the internal system through various boundary buses. There is no need for system stability considerations to define the external and internal systems, since the decomposition technique is just used as a mathematical tool for efficient solution.

The decomposition technique finds application in three instances:

1 - To solve large problems on uniprocessor computers with reduced core capacity. The decomposition technique is then used to break apart the complete power system into smaller subsystems which are individually solved.

2 - To save computation time on a uniprocessor computer when using the frequency response algorithm described in [1]. In this case it is assumed that the applied disturbance and monitored outputs are all located within the internal system [2].

3 - To speed up the solution of power system small-signal stability problems through use of parallel computation [3].

The power of the algorithms proposed in [1] rests in the fact that the electric network equations are expressed in their unreduced form. This fact also accounts for the efficiency of the decomposition technique, which is similar and equally as flexible as those proposed in [4,5] for the solution of steady-state power system problems.

II. MATHEMATICAL FORMULATION

Reference 1 presents efficient algorithms for the solution of small-signal stability problems of large power systems (see Appendix). All these algorithms involve the solution of an equation of the general form:

$$\begin{bmatrix} J_A' & J_B \\ J_C & J_D \end{bmatrix} \cdot \underline{X} = \underline{b} \tag{1}$$

where J_A' corresponds to the matrix block J_A of Figure 1 with some extra terms added to its diagonal. The characteristics of the Jacobian matrix shown in Figure 1 are described in the Appendix.

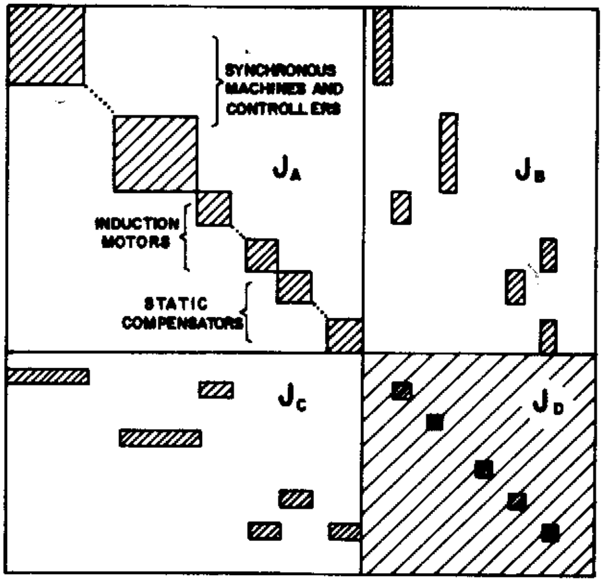


Figure 1 : The Power System Jacobian Matrix (The hatched blocks contain non-zero elements and have sparse structure)

Consider an interconnected system which is to be decomposed into two subsystems. By reordering the variables of equation (1) one obtains:

$$
\begin{bmatrix} A_E & & B_E & & \\ & A_I & & & B_I \\ C_E & & Y_{EE} & Y_{EB} & \\ & & Y_{BE} & Y_{BB} & Y_{BI} \\ & C_I & & Y_{IB} & Y_{II} \end{bmatrix} \cdot \begin{bmatrix} \underline{X}_E \\ \underline{X}_I \\ \underline{V}_E \\ \underline{V}_B \\ \underline{V}_I \end{bmatrix} = \begin{bmatrix} \underline{b}_E \\ \underline{b}_I \\ \\ \\ \\ \end{bmatrix} \qquad (2)
$$

where subscripts "E", "I" and "B" refer to variables or matrix blocks associated with the external system,internal system and boundary buses, respectively. All variables in (2) are incremental values, but the symbol Δ is omitted for simplicity.

The correspondence between blocks of (1) and (2) is as follows:

$$
J'_A = \begin{bmatrix} A_E & \\ & A_I \end{bmatrix} \qquad J_B = \begin{bmatrix} B_E & & \\ & & B_I \end{bmatrix}
$$

$$
J_C = \begin{bmatrix} C_E & \\ & \\ & C_I \end{bmatrix} \qquad J_D = \begin{bmatrix} Y_{EE} & Y_{EB} & \\ Y_{BE} & Y_{BB} & Y_{BI} \\ & Y_{IB} & Y_{II} \end{bmatrix}
$$

Elimination of the external system variables ($\underline{X}_E$, $\underline{V}_E$) and independent vector $\underline{b}_E$ is then performed, yielding:

$$
\begin{bmatrix} A_I & & B_I \\ & Y^*_{BB} & Y_{BI} \\ C_I & Y_{IB} & Y_{II} \end{bmatrix} \cdot \begin{bmatrix} \underline{X}_I \\ \underline{V}_B \\ \underline{V}_I \end{bmatrix} = \begin{bmatrix} \underline{b}_I \\ \underline{b}^*_B \\ \\ \end{bmatrix} \qquad (3)
$$

where: $Y^*_{BB} = Y_{BB} - Y_{BE}\,(Y^*_{EE})^{-1}\,Y_{EB}$

$Y^*_{EE} = Y_{EE} - C_E\,(A_E)^{-1}\,B_E$

$\underline{b}^*_B = -Y_{BE}\,(Y^*_{EE})^{-1}\,\underline{b}^*_E$

$\underline{b}^*_E = -C_E\,(A_E)^{-1}\,\underline{b}_E$

Subvectors $\underline{X}_I$, $\underline{V}_B$ and $\underline{V}_I$ can be solved by using either equation (2) or (3). The solution vector $\underline{X}$, formed by subvectors $\underline{X}_E$, $\underline{X}_I$, $\underline{V}_E$, $\underline{V}_B$ and $\underline{V}_I$, can also be obtained by decomposing (2) into two sets of equations:

$$
\begin{bmatrix} A_E & B_E & \\ C_E & Y_{EE} & Y_{EB} \\ & Y_{BE} & Y^{ext}_{BB} \end{bmatrix} \cdot \begin{bmatrix} \underline{X}_E \\ \underline{V}_E \\ \underline{V}_B \end{bmatrix} = \begin{bmatrix} \underline{b}_E \\ \\ \\ \end{bmatrix} \qquad (4a)
$$

$$
\begin{bmatrix} A_I & & B_I \\ & Y^{int}_{BB} & Y_{BI} \\ C_I & Y_{IB} & Y_{II} \end{bmatrix} \cdot \begin{bmatrix} \underline{X}_I \\ \underline{V}_B \\ \underline{V}_I \end{bmatrix} = \begin{bmatrix} \underline{b}_I \\ \\ \\ \end{bmatrix} \qquad (4b)
$$

where: $Y_{BB} = Y^{int}_{BB} + Y^{ext}_{BB}$ (5)

Elimination of $\underline{X}_E$ and $\underline{V}_E$ in (4a) yields:

$$
(Y^{ext}_{BB})^*\ \underline{V}_B = \underline{b}^*_B \qquad (6)
$$

where: $(Y^{ext}_{BB})^* = Y^{ext}_{BB} - Y_{BE}\,(Y_{EE})^{-1}\,Y_{EB}$

By adding (6) to the corresponding blocks in (4b) one obtains equation (3). The variables of the internal system and boundary buses are calculated from equation (3). From knowledge of $\underline{V}_B$, the vector of boundary bus voltages, the external system variables are solved by using a modified form of equation (4a):

$$
\begin{bmatrix} A_E & B_E \\ C_E & Y_{EE} \end{bmatrix} \cdot \begin{bmatrix} \underline{X}_E \\ \underline{V}_E \end{bmatrix} = \begin{bmatrix} \underline{b}_E \\ -\,Y_{EB}\ \underline{V}_B \end{bmatrix} \qquad (7)
$$

The matrices of equations (2), (3), (4) and (7) are highly sparse (see Appendix) and the decomposition technique is computationally efficient with the use of sparsity-oriented routines for Gaussian Elimination.

Note that the equations shown in this Section can easily be generalized for the case of multiple external systems, but this is here omitted for brevity. A similar formulation, devoted to steady-state power system analysis, can be found in [5].

It should be emphasized that the decomposition technique does not introduce any potential numerical stability problems into the basic algorithms reported in [1]. The use of equations (2) or (4a) and (4b) lead to the same numerical values for the solution vector. The difference in round-off errors between the two cases is insignificant to any engineering application. Thefore there is no degradation in the convergence rate of the algorithms presented in [1] when using the proposed decomposition technique.

III. CONSIDERATIONS ON THE DECOMPOSITION PRINCIPLE

Consider the electric system shown in Figure 2a which comprises an internal system and three interconnected external systems. Buses B1, B2 and B3 define the internal system boundary while buses B12, B13 and B23 represent the boundaries between external systems. In order to apply the decomposition methodology described in Section II it is necessary to define an Extended Internal System comprising the original internal system plus all the boundary buses in the complete system. Connectivity of the Extended Internal System comes through the elements or "lines" generated by Gaussian Elimination of all external system variables, as shown in Figure 2b. The equivalent shunts and injected "currents" at the boundary buses, generated by the aforementioned elimination, are omitted in the figure for simplicity.

Sparsity Aspects

The Jacobian matrix of each external system is formed with its network equations optimally ordered per Tinney-2 scheme [6]. In the Jacobian matrix formation of the Extended Internal System, the Tinney-2 ordering scheme is carried out considering the external system equivalents plus the Extended Internal System (see Figure 2b).

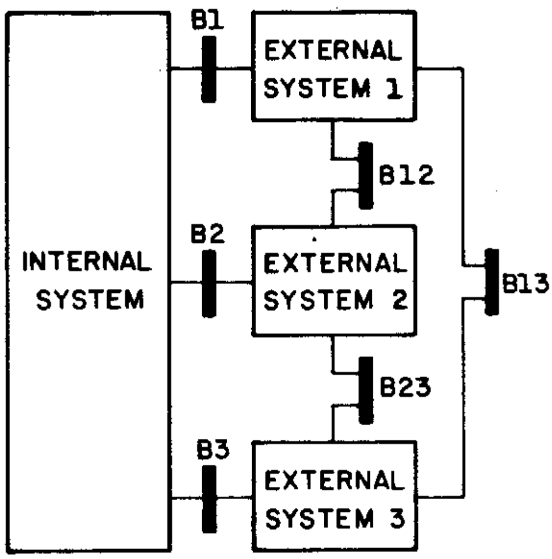


FIG. 2a

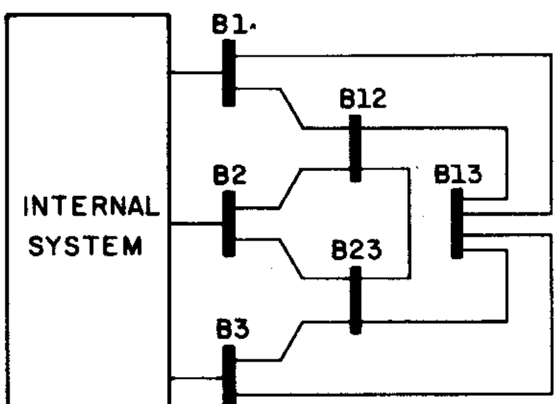


FIG. 2b

Figure 2 : (a) Diagram of Interconnected System
(b) Extended Internal System plus External Equivalents (The equivalent shunts at the buses are omitted)

It may be convenient to retain selected buses in order to preserve the sparsity of the external system equivalents [6]. The decomposition technique can consider this by simply including the retained external buses in the boundary bus partition $\underline{V}_B$ of the Extended Internal System.

It is interesting to note that the Extended Internal System can be comprised of only a set of non-connected buses and does not need to have components with dynamic representation. Let us again consider the system of Figure 2 and define the depicted internal system as the fourth external system and the set of six boundary buses as the new Extended Internal System. The solution of the Extended Internal System variables (stage 4 of the computational scheme presented in Figure 3) involves the factorization of matrix Y^*_{BB}, which is here associated with a system of six buses and 11 equivalent lines (see Section II).

Suggested Implementation on a Multiprocessor Network

The decomposition technique presented in this paper allows efficient implementation on a parallel multiprocessor network, in which every CPU would specifically solve a designated subsystem. Thus it is convenient to break apart the interconnected system into a number of subsystems of approximately the same size. There is no need for system stability considerations to define these subsystems, since the decomposition technique is just used as a mathematical tool for efficient parallel solution.

Figure 3 shows in a schematic way the implementation of the AESOPS algorithm [1,7,8] on a parallel multiprocessor network. The blocks identified as ES_i and IS represent different processors solving the "i-th" External System and the Extended Internal System respectively. It is seen that this parallel computation scheme has a bottleneck associated with the solution for the Extended Internal System variables. Thus it is important to select an adequate set of boundary buses so as to preserve sparsity in the Extended Internal System matrix, and also to keep it as small as possible.

The system decomposition principle can be efficiently implemented on a simple multiprocessor structure: independent processors connected on a single bus. There will not exist problems of high comunication overhead as most of the computational work is independently done by the various processors in parallel. As seen in Figure 3, there is no data transmission between processors which are in parallel.

Speed Up Obtained With The Parallel Computation Scheme

Some figures are now given on the speed up obtained by using the decomposition technique on a parallel computation scheme. The major part of the computational work in the implementation shown in Figure 3 is devoted to the execution of steps 3, 4 and 5. The computational effort for sparsity-oriented factorization and solution will be considered to vary linearly with matrix size in this analysis [6] . Accordingly, on ideal conditions the implementation shown in Figure 3 approximately solves each one of steps 3 and 4 in a time t/ns. Step 5 takes about t/(4ns) since it only involves forward and back substitutions. All subsystems are assumed to be of the same size and "ns" is the number of subsystems in which the interconnected system was decomposed. The constant "t" is the time needed for sequential solution of the complete interconnected system on a single processor. For the case of 50 subsystems of equal size and the same number of independent processors, the maximum speed up obtained from the use of the described parallel computation scheme would be of the order of 20 times.

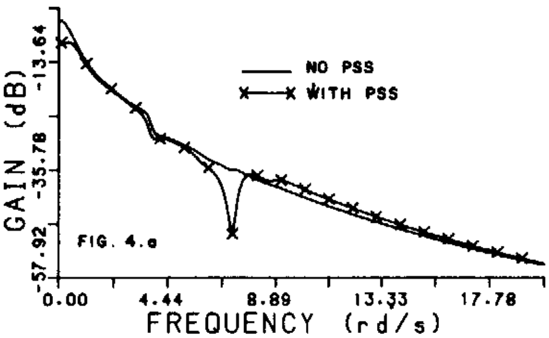
The task of decomposing the interconnected system into a few subsystems can easily be done according to geographical units such as the individual utilities, "areas" and "subareas". However, the automatic decomposition into a large number of subsystems for maximum efficiency of the described parallel computation scheme would require very clever bus ordering algorithms of the block-bordered diagonal form (BBDF) [4,5].

IV. RESULTS

Frequency Response Calculations

The decomposition technique applied to the frequency response algorithm [1] , on a uniprocessor computer, can significantly reduce computational work. It is recommended for use in engineering studies in which the applied disturbance and monitored outputs are all located within the internal system. Subvector $\underline{b}_E$ of (2) does not exist in this application and, accordingly, $\underline{b}^*_B$ of (3) is a null vector. The external system frequency response contributions are previously calculated and stored on disk files for every discrete value of applied frequency within the range of interest. A typical study case could have a few generating plants as the internal system and the whole interconnected system as the external system. In this application, the operating point or the topology of the internal system can even be changed as long as the conditions at the boundary buses remain the same.

Results are shown for the well-known New England test system [7], whose Internal System is here defined as comprising the generators at buses 30 and 38 (refer to the one-line diagram shown in [7]). Only these two buses belong to the Internal System which is therefore non-connected. The External System comprises the other eight generators and the complete electric network apart from buses 30 and 38. The Bode plots for the transfer function $V_t(s)/V_{ref}(s)$ for the two generators of the Internal System are shown in Figure 4 for four cases, which were obtained by considering or not the presence of power system stabilizers. These four plots were obtained using a unique data file, containing the previously calculated frequency response contributions of the External System. The computation time required to produce these four plots was reduced by half through use of the decomposition technique.



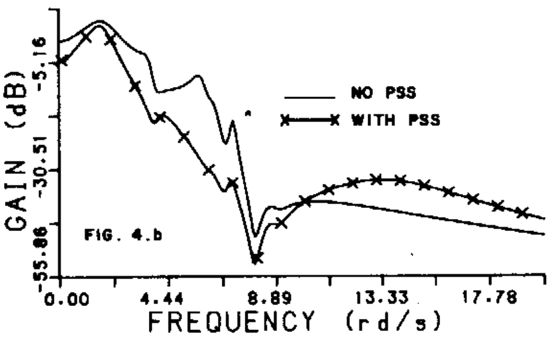


Figure 4 - Bode Plots of $V_t(s)/V_{ref}(s)$

(a) For Generator at Bus nº 30
(b) For Generator at Bus nº 38

Eigenvalue Calculations in a Parallel Mode

A computer program was developed on a uniprocessor computer to simulate a parallel multiprocessor network in the solution of the AESOPS and Implicit Inverse Iteration algorithms. The structure of the original code [1] remained basically the same but some additional subroutines were needed to allow application of the decomposition technique. The simulation program solves one subsystem at a time and store partial solutions on disk files, which are retrieved when needed. Parallel computer CPU time is considered as being the time needed for the uniprocessor solution of the largest subsystem within each parallel path (see Figure 3).

Simulation tests were carried out on the South-Southeast Brazilian System represented by 356 buses, 628 lines and 49 generators. The interconnected system was decomposed into five subsystems, whose dimensions are given in Table 1, according to natural geografic groupings of elements.

SUBSYSTEMS

	ES1	ES2	ES3	ES4	IS
Buses	105	43	40	30	138
Generators	14	6	8	1	20
Boundary Buses	4	4	4	3	15

Table 1: Subsystem Dimensions in Test System.
ES_i = i-th External System; IS = Internal System

The simulation program is still at prototype level: the code needs to be further optimized and it only allows a maximum of four boundary buses per external system. As a consequence, only a low number of subsystems of very different sizes could be generated (see Table 1). This precluded a correct evaluation of the speed up which can be obtained by the parallel computation scheme.

The eigenvalue pair associated with an inter-area mode of oscillation, in which the whole Southeast System swings against the South System, is $\lambda = -0.113 \pm j\,3.403$. The small-signal stability program of [1] was used to find this eigenvalue by the Implicit Inverse Iteration (III) algorithm starting from an estimate $\lambda = 0 + j3$. The III algorithm converged to a tolerance of 10^{-6} in 11 iterations in 51 seconds of CPU on a VAX-11/780 computer. The simulation program was run for four cases in which the system was divided into 2,3,4 or 5 subsystems. In all cases the eigenvalue results were the same (up to 7 significant figures) as that obtained with the original non-partitioned system. The simulation program required 100 seconds of CPU to run the case with 5 subsystems, due to the read/write disk operations during the convergence process and the less efficient code.

One natural advantage of the parallel computation simulation program is that it can solve problems "ns" times larger than the memory capacity of the uniprocessor computer ("ns" is the number of subsystems in which the system is divided).

V. CONCLUSIONS

The decomposition technique is very effective in saving computation on a uniprocessor computer when conducting frequency response studies on multimachine systems.

The suggested decompositon technique applied to the algorithms of [1] makes possible the parallel computation of small-signal stability problems. In general, the larger the number of subsystems in which the system is torn, the faster the desired calculation.

The parallel computation of eigenvalues, via AESOPS and Implicit Inverse Iteration algorithms [1], was simulated on a uniprocessor computer. Results were obtained for a medium-size interconnected system which was divided into five subsystems according to natural geographical units, such as "areas" and "subareas". Further partition of the system was not attempted due to present limitations of the prototype version of the program.This precluded an evaluation of the speed up which can be obtained by parallel computation.

It was observed that there is no degradation in the results or convergence rate of the algorithms presented in [1] when using the decomposition technique.

Efficient algorithms are needed to automatically decompose a given power system into many subsystems and in the most convenient form for parallel computation.

Diakoptics [9] could also have been used in connection with the algorithms of [1],but the decomposition technique here described leads to simpler computer implementation.

FIGURE 3

AESOPS ALGORITHM IMPLEMENTED ON A PARALLEL MULTIPROCESSOR NETWORK

(1) Complete system is broken apart into various subsystems.

(2) Form Jacobian matrix for all subsystems.

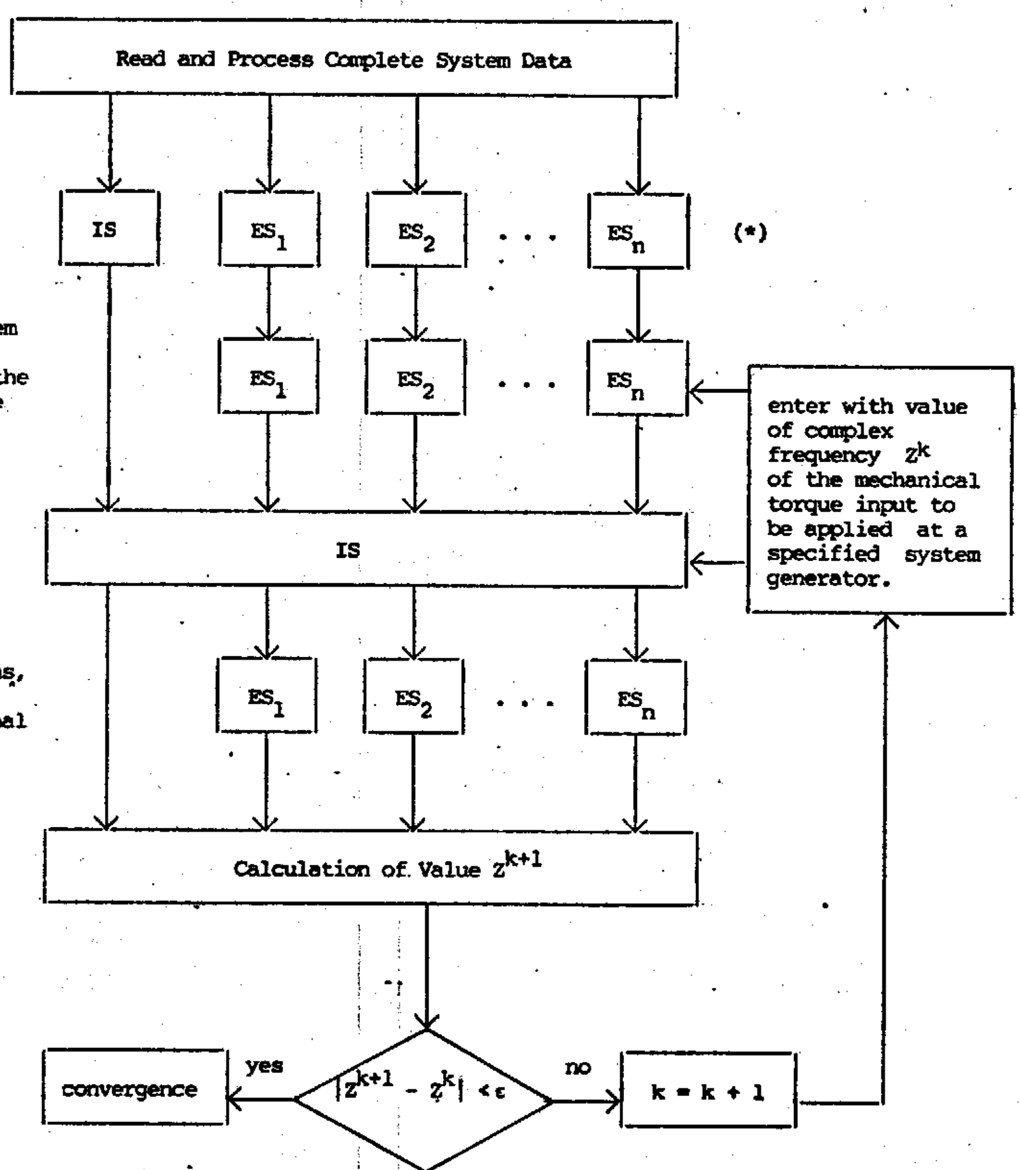
(3) Compute external system equivalents and the injected currents at the boundary buses for the given perturbation frequency Z^k.

(4) Compute variables for the internal system after incorporating all external systems contributions.

(5) With knowledge of the boundary bus conditions, compute the variables for the various external systems.

(6) The solution for the complete system has been obtained, and the new perturbation frequency Z^{k+1} can be calculated.

(7) At convergence, the desired eigenvalue is $\lambda = Z^{k+1}$, where k is the iteration number.



(*) The blocks identified as ES_i and IS represent different processors solving the "i-th" external system and the Extended Internal System respectively.

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APPENDIX

EFFICIENT ALGORITHMS FOR THE SOLUTION OF SMALL-SIGNAL STABILITY PROBLEMS

The package of computer programs developed in the work reported in [1] contains efficient algorithms for the calculation of:

1 - Exact eigenvalue closest to a specified point in a complex plane, and associated eigenvector (implicit inverse iteration algorithm).

2 - Exact eigenvalues associated with the dominant modes of oscillation of the system (AESOPS algorithm) [7, 8] .

3 - Frequency response plots of transfer functions between any two specified variables in the system.

4 - Step response results obtained via implicit trapezoidal integration formula. This algorithm was not described in [1] for being of straightforward implementation.

All the aforementioned functions of the program package make use of the Jacobian matrix of the differencial-algebraic set of equations for the electric system , calculated for a system operating point. All the pertinent formulation has been described in [1] . The Jacobian matrix, whose structure is depicted in Figure 1, caters for various models of synchronous generators and associated controllers, induction motors, non-linear loads of different characteristics and static VAR compensators. Special routines also allow the user to specify power system controllers without restriction on their order or topology.

The Jacobian matrix is formed by first entering the blocks of equations for every system generator. After this the induction motor blocks are built and are followed by the static VAR compensator blocks. The network equations come last in the Jacobian matrix formation. Submatrix J_D, shown in Figure 1, is practically equal to the nodal admittance matrix of the network, expanded into its real and imaginary parts. The only difference lies in the fact that the (2x2) diagonal bloks of J_D corresponding to the nodes which contain linear, non-linear or induction motor loads, have extra partial derivative terms added to them.

The power system Jacobian matrix is extremely sparse and of very high order, and the full use of sparsity techniques becomes imperative. As seen from Figure 1, there will be practically no fill-in during the LU factorization of the submatrix J_A and therefore the ordering of its equations is not critical [1]. The network equations of submatrix J_D are optimally ordered per Tinney-2 scheme which selects the node for elimination at each stage that introduces the fewest non-zero elements [6] .