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ANALYSIS OF LOW-DAMPED ELECTROMECHANICAL OSCILLATIONS IN LARGE POWER SYSTEMS

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Résumé

Frequency response and eigenvalue techniques are described for the analysis of small signal stability of multimachine power systems. A highly efficient algorithm is presented for the exact calculation of eigenvalues and eigenvectors for very large power systems. Stabilizer gain margins, as affected by the addition of power system stabilizers to other generators in the system, are evaluated.

Keywords

Damping - Electromechanical Oscillations - Excitation Control - Stability - Eigenvalues - Frequency Response

1. Introduction

The predominant hydrogeneration and the associated long transmission lines of the Brazilian power systems constitute a source of potential problems for low damped electromechanical oscillations. A great deal of attention has been given to the study of this problem in this country and a good number of generating plants already incorporate power system stabilizers or will have them installed in the near future. This paper describes the preliminary results obtained from a research work being carried out in this field.

A large amount of work has been done throughout the world in the area of small signal dynamics of power systems. The accumulated knowledge on the mechanisms leading to the build-up of undamped oscillations and also on the ways to prevent these problems via control methods is well developed [1,2,3,4].

The computation of the eigenvalues of the multimachine system state matrix through the QR transformation method is an effective analysis tool but can not handle efficiently systems with more than 200 state variables [5]. This limiting factor generates continuous research efforts on ways to obtain eigenvalues of power systems of much larger order. In this paper a highly efficient technique is presented for the exact calculation of eigenvalues and eigenvectors for very large power systems, which has low computer memory requirements. This method requires the formation of a very large and sparse system Jacobian, and the preliminary results obtained indicate that this technique will prove valuable for the analysis of low damped electromechanical oscillations of interconnected systems.

Frequency response analysis can readily be performed using a variation of the eigenvalue algorithm and were used in this work for stabilizer tuning in the multimachine environment. This technique was also used in identifying the best locations in the system for placing the damping effort and in evaluating power system stabilizer gain margins as affected by the presence of other stabilizers in the system.

2. State Space System Models

Two efficient multimachine state matrix formulations were developed. The first caters for different synchronous generators models and their controllers, while the other also incorporates models for induction motors, non-linear loads, static VAR compensators and generator incremental saturation. Sparsity techniques are fully used to ensure efficient computation of the state matrix while keeping the computer core requirements to the minimum. The more elaborate formulation is here described as it is needed in the implementation of the proposed eigenvalue algorithm.

The eigenvalue and the transient stability programs have a common input data format, which is a very desirable feature as the system eigensolution may be easily obtained before transient stability runs are made [1].

The power system stability problem can be represented by an algebraic-differential set of equations. The linearized system state matrix is derived from the Jacobian of the entire set of equations evaluated at an operating point [6]:

$$\begin{bmatrix} \Delta\dot{\underline{X}} \\ \underline{0} \end{bmatrix} = \begin{bmatrix} J_a & J_b \\ J_c & J_d \end{bmatrix} \begin{bmatrix} \Delta\underline{X} \\ \Delta\underline{Y} \end{bmatrix} \qquad (1)$$

which is then reduced to the state variables only: $\Delta\dot{X} = AX$, where $A = J_a - J_b \cdot J_d^{-1} \cdot J_c$. The matrix A eigenvalues determine the singular point stability of the non-linear system.

The two state matrix formulations developed allow easy implementation of routines for the computation of eigenvalue sensitivity coefficients. The experience gained from the use of eigenvalue sensitivity routines is summarized in Section 5 of this paper.

3. An Efficient Eigenvalue Solution Algorithm

The inverse iteration method is mainly used for the calculation of eigenvectors given a good approximation of an eigenvalue [7]. It can also be used for finding the eigenvalue which is the closest to a point in the complex plane and its respective eigenvector. The basic inverse iteration algorithm can be described by:

$$(A - qI)\, \underline{W}_{k+1} = \underline{Z}_k$$

$$\underline{Z}_{k+1} = \frac{\underline{W}_{k+1}}{\max(\underline{W}_{k+1})} \qquad (2)$$

where 'k' is the iteration number, 'I' is the identity matrix, 'q' the approximation of the desired eigenvalue and $\max(\underline{W}_{k+1})$ is the element of largest magnitude in this vector. The vector $\underline{Z}_k$ which has an arbitrary initial value, corresponds to the desired eigenvector at convergence.

Multimachine state matrices are asymmetrical and not sparse, and the percentage of non-zero elements usually ranges from 25 per cent to a maximum of 66 per cent depending on the degree of system modelling. This fact can be visualized by noting that current, voltage and electrical power at any generator terminal must be expressed as an explicit function of the internal voltages and rotor angles of every other generator in the system.

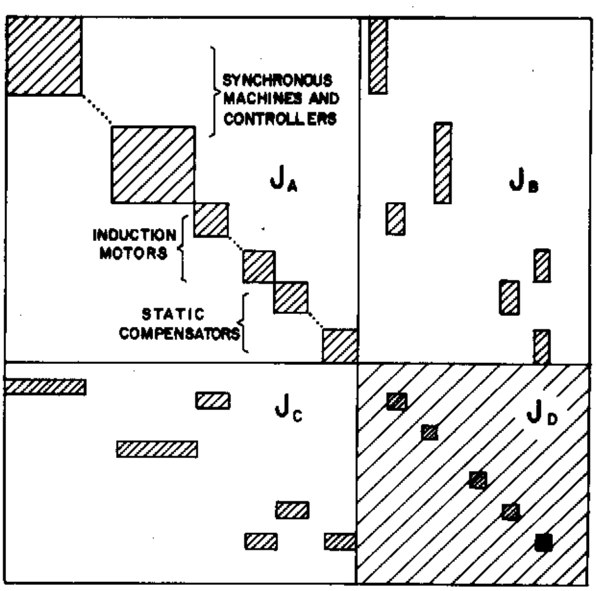

Figure 1 :Power System Jacobian
The hatched areas indicate sparse blocks

The implementation of the inverse iteration method in the form described in equation (2) is impractical for large systems since the matrix A is not sparse. This method can however be implemented in a very efficient form [8] if it is applied directly to the Jacobian of equation (1), which is very sparse.

$$\begin{bmatrix} J'_a & J_b \\ J_c & J_d \end{bmatrix} \begin{bmatrix} \underline{W}_{k+1} \\ \underline{R} \end{bmatrix} = \begin{bmatrix} \underline{Z}_k \\ \underline{0} \end{bmatrix} \qquad (3)$$

where $J'_a = (J_a - qI)$, and $\underline{R}$ is a vector of intermediate results.

An important point as regards equation (3) is that submatrix J_a should be triangulated first in the matrix factorization, since non-sparse LU factors would be obtained otherwise. In order to have a more efficient solution it is necessary to adopt the same scheme used for the simultaneous solution of the transient stability equations [9]. This involves grouping all differential and algebraic equations for the various generators into separate blocks so as to have only the network equations on the lower part of the Jacobian (Figure 1). Note that submatrix J_D differs from the real (2nk x 2nk) nodal admittance matrix by the (2x2) diagonal blocks corresponding to the terms for the linear, non-linear and induction motor loads.

The matrix equation for the inverse iteration algorithm is now given by

$$\begin{bmatrix} J'_A & J_B \\ J_C & J_D \end{bmatrix} \begin{bmatrix} \underline{W}_{k+1} \\ \underline{Q} \end{bmatrix} = \begin{bmatrix} \underline{Z}^e_k \\ \underline{0} \end{bmatrix} \qquad (4)$$

where $\underline{W}^e_{k+1} = \begin{bmatrix} \underline{W}_1 \\ \underline{U}_1 \\ \vdots \\ \underline{W}_{nt} \\ \underline{U}_{nt} \end{bmatrix}_{k+1}$, $\underline{Z}^e_k = \begin{bmatrix} \underline{z}_1 \\ \underline{0} \\ \vdots \\ \underline{z}_{nt} \\ \underline{0} \end{bmatrix}_k$, $\underline{W}_{k+1} = \begin{bmatrix} \underline{W}_1 \\ \underline{W}_2 \\ \vdots \\ \underline{W}_{nt} \end{bmatrix}_{k+1}$

and $J'_A = (J_A - qL)$ where L is a diagonal matrix with unitary values at the state variable rows and zero at the others. The subscript 'nt' corresponds to the total number of generators, induction motors and static VAR compensators in the system. The vector $\underline{W}_{k+1}$ is contained in $\underline{W}^e_{k+1}$ and at convergence corresponds to the desired eigenvector. The vectors $\underline{Q}$ and $\underline{U}_i$ contain intermediate results.

The submatrix J_D has the same sparsity structure as the nodal admittance matrix and should be ordered in a way to minimize fill-in during factorization. Equation (4) should be solved in a partitioned manner for higher efficiency [9,10]:

$$J^*_D\, \underline{Q} = -J_C\, (J'_A)^{-1}\, \underline{Z}^e_k$$

$$J'_A\, \underline{W}^e_{k+1} = \underline{Z}^e_k - J_B \underline{Q} \qquad (5)$$

where $J^*_D = J_D - J_C\, (J'_A)^{-1}\, J_B$

The matrix $J_C (J_A^i)^{-1} J_B$ is block diagonal and each (2x2) diagonal block can be obtained from the product $J_C^i (J_A^i)^{-1} J_B^i$ where J_A^i, J_B^i and J_C^i are blocks associated to the "i-th" system component (generator, induction motor or static compensator). Note that each J_A^i can be factorized separately, which allows large savings in core requirements and solution time. The transpose eigenvector, needed for eigenvalue sensitivity calculations, can be obtained using the same LU factors of J_A' and J_D^* by noting that $A^t = U^t.L^t$.

4. Frequency Response Analysis of Large Power Systems

Frequency response analysis of very large order systems have been performed for eigenvalue estimation [11] and in the evaluation of subsynchronous oscillatory stability [12]. A common feature to the two methods above is that a state matrix description of the system is avoided in order to improve computational efficiency. The linear incremental models for the network and generators are kept apart and a combined solution is obtained at low cost for discrete values of frequency in the range of interest.

The system transfer function matrix, obtained from the state space description, is given by $F(s) = C(sI-A)^{-1} B+D$. A variation of the proposed algorithm for inverse iteration (Section 3) can be used to efficiently obtain the system matrix for discrete values of frequency and, though less efficient than the methods of [11,12], leads to a very flexible computer implementation.

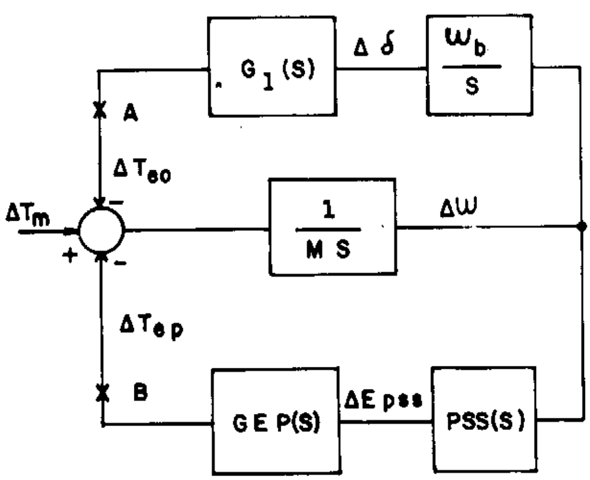

Figure 2 : Transfer Function Model for Generator and Excitation System - Torque-Angle Loop Representation

Frequency response methods allow much deeper insight into small signal dynamics than time response methods, and have widespread use in power system controller design. In studies of stabilizer tuning the power system is generally represented by a single generator connected to a Thevenin equivalent or by a low order equivalent model derived from frequency response measurements at the field.

This paper reports on the experience obtained with a computer program used for small signal stability assessment and stabilizer tuning, which produces frequency response plots considering the dynamics of the entire multimachine system. Such a detailed program is not generally necessary but is very useful in the analysis of some difficult cases. It is also an important instrument in the evaluation of the various simplifying assumptions usually adopted in this kind of study.

Both torque-angle loop and exciter loop analyses are useful in stabilizer tuning methods, though the latter due to excitation system flexibility is normally used in field measurement tests [3]. The stabilizer gain and phase margins can be obtained from frequency response analysis applied to the torque-angle loop broken at point B (Figure 2) or to the exciter loop broken at point D (Figure 3), but only the latter can be directly measured in the field.

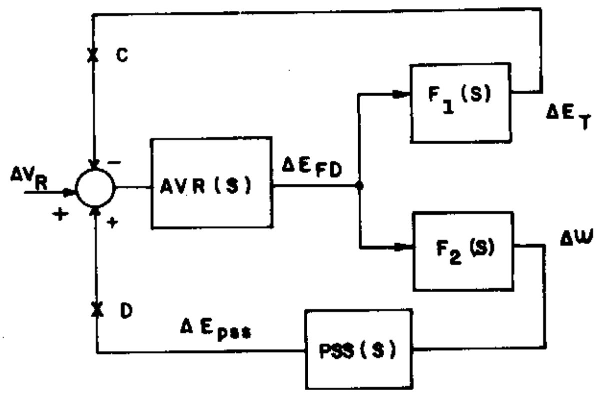

Figure 3 : Transfer Function Model for Generator and Excitation System - Exciter Loop Representation

Power system stabilizer (PSS) tuning is generally carried out by determining the phase lag of GEP(s) (Figure 2) at the local and inter-area modes and providing a phase lead (and adequate gain) through the stabilizer in order to produce electrical torque in phase with speed [3]. The computer program developed can calculate GEP(s) considering the dynamics of the entire multimachine system.

A few points of interest are worth mention regarding the exciter-loop analysis (Figure 3):

a) By breaking simultaneously the loop at points C and D, the resulting open loop transfer function has the automatic voltage regulator (AVR) as an explicit function. As a consequence, AVR gain and phase margins can readily be obtained [13].
b) By breaking the loop at point D, the gain and phase margins of the stabilizer can be obtained. A generally adequate PSS design for local mode damping can be obtained by applying Nyquist criterion to this open loop transfer function.
c) The excitation system closed loop frequency response plots $\Delta E_T(s)/\Delta V_R(s)$, with and without the PSS, can be compared with field tests in order to verify the adequacy of the models used in the studies. They also help to confirm the effectiveness of the PSS in improving system performance [14].

The multimachine system closed loop transfer function is described in Figure 4, where only the exciter loops for the various generators are explicitly shown. The major part of the computational effort in the method here described concerns the calculation, for every discrete value of frequency in the range of interest, of the system matrix G. Therefore, the set of matrices G(jw) for the different frequencies are stored in disc and are used as many times as necessary while the designer decides on the L matrix which appears to give the best results, or determines the gain margins for the stabilizers in the system. The

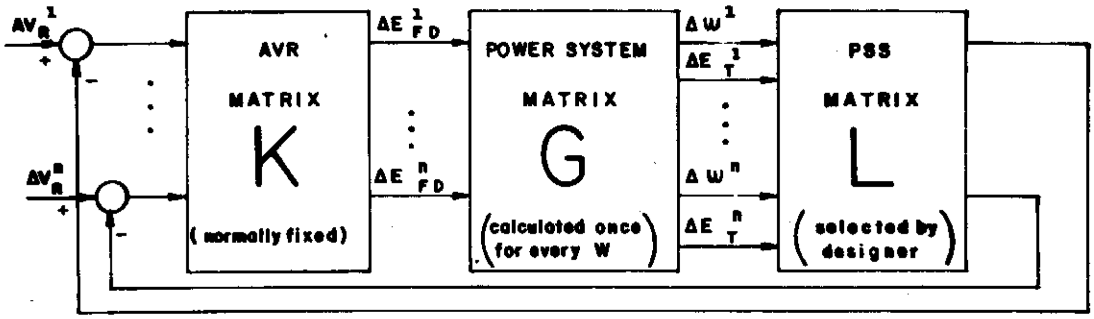

Figure 4 :Power System Closed Loop Transfer Function Model

redundancy in the rotor angle states is removed before obtaining the system matrix G in order to eliminate the open loop transfer function pole at the origin.

The multimachine system closed loop transfer function is given by LGK(s)/(I+LGK(s))and system stability can be determined by applying the standard Nyquist criterion to the determinant of (I+LGK(s)) [13]. In the work here described the single-input-single-output Nyquist criterion is used to determine system stability and to design power system stabilizers. This is carried out by opening the i-th generator exciter loop at points C and D (See Figure 2), while keeping all the other exciter loops closed. The dynamic effects of the controller loops for all the other generators are reflected in the transfer functions relating the "i-th" generator variables, through a Gaussian elimination process. The final results are the values, for the specific frequency signal applied, of the transfer functions $\Delta E_T^i(s)/\Delta E_{FD}^i(s)$ and $\Delta W^i(s)/\Delta E_{FD}^i(s)$ which can then be connected and analysed in the three different ways already described. It should be noted that as generator 'i' is generic, AVR and PSS gain and phase margins may be obtained for every generator in the system.

5. Eigenvalue Studies

Some general conclusions drawn from eigenvalue analysis are here briefly described. The results obtained from sensitivity studies showed that first order eigenvalue sensitivity is a very desirable analysis tool for large systems while second order sensitivity coefficients were found to be completely inadequate. Actually, in our large system studies second order sensitivity coefficients always led to worse eigenvalue estimates even for increments as small as .1 per cent in certain system parameters. The evidence is that there is simply no point in using second order terms in the Taylor expansion if for practical parameter increments the series is not monotonically convergent.

Induction motors, when compared with linear impedance loads, were found to have a stabilizing effect on synchronous generator hunting modes and this is more prominent in the absence of automatic excitation. The effects of motor inertia were found to be relevant but the load torque characteristic had negligible influence on system small signal dynamics.

Saturated generators operating in the over-excited region were found to have stability limits about 20 to 25 per cent greater than those of the unsaturated machines, these results being in line with those of Crary [15]. On the other hand, generator incremental saturation effects proved minimal when automatic excitation was incorporated to the system, reinforcing the results obtained in [16]. Therefore, the importance given in some publications to the representation of generator incremental saturation in the study of regulated power systems is not justified according to the results obtained.

6. Frequency Response System Studies

6.1 Single machine infinite bus system

A single machine system is discussed here where a local load is supplied by the generator and by the infinite bus through a high impedance transmission line (same as system B of Reference [4]. An eigenvalue analysis of this system showed it to have a very low damped electromechanical mode which is unaffected by variations in the AVR gain. The analysis of GEP(s) revealed that no phase advance was necessary for the power system stabilizer and the Nyquist criterion applied to the system excitation loop confirmed this fact. Considering the PSS transfer function as a pure gain, the root-locus analysis showed the undamped pole moving towards the left plane, as the gain increased, with no change in its imaginary component. This denoted that PSS action was providing pure damping torque to this system and therefore, the frequency of oscillation did not alter [3]. An interesting observation was that the instability gain for the rotor speed input stabilizer was more than ten times larger than an approximatly optimum value of gain. Therefore, one may conclude that the recommended practice of setting the stabilizer gain to a third of the instability value does not apply in all cases.

6.2 Nine machine system

Studies were carried out on a real nine machine system which presented low damped oscillations, and the results obtained checked with those of the eigenvalue analysis. The Nyquist plots $\Delta E_T^i(jw)/\Delta V_R^i(jw)$, i=1, ..., 9, were calculated for a particular loading condition and indicated that only for one generator instability could appear by raising the AVR gain. The very same information could however be obtained using a single machine system representation: the particular generator against the equivalent Thevenin for the whole system.

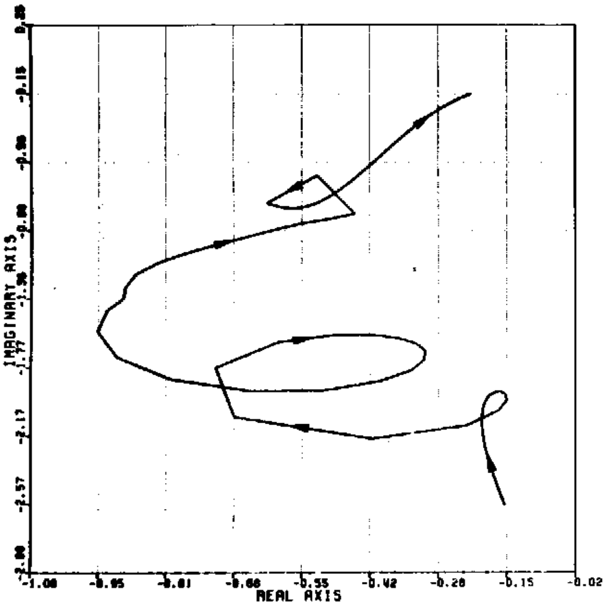

Figure 5 :Nyquist Plot $\Delta E_T(s)/\Delta V_R(s)$ for Generator No. 8 - All Generators without Power System Stabilizers

Frequency response polar plots can give indication of the presence of low damped modes of oscillation in large power systems [12]. Figure 5 shows a Nyquist plot $\Delta E_T(jw)/\Delta V_R(jw)$ for a particular generator which indicates the presence of two low damped stable modes and of an unstable mode, the latter being identified by the fast counter-clockwise changes in the plot. These observations were confirmed by eigenvalue analysis and these troublesome poles are: $- .209 \pm j8.03$, $- .167 \pm j9.71$ e $+ .110 \pm j11.6$.

In the present analysis the denominator of the open loop transfer function (characteristic polynomial) changes when the observation point is moved from one generator to another. This fact may be used to advantage in order to obtain additional information on the complex dynamic interactions of the system. The closed loop transfer function, seen from any generator in the system has obviously the same characteristic polynomial, but due to pole-zero cancellation a system low damped mode may not be observed from a particular generator. The fact that a low damped mode is clearly observed in a Nyquist curve for a particular generator may be an evidence that stabilizing action at this generator will have an effect on the damping of this mode.

The plot shown in Figure 6 differs from that of Figure 5 by the fact that power system stabilizers have been added to four other generators in the system. The correct location and tuning of the stabilizers damped out the three troublesome modes of the system.

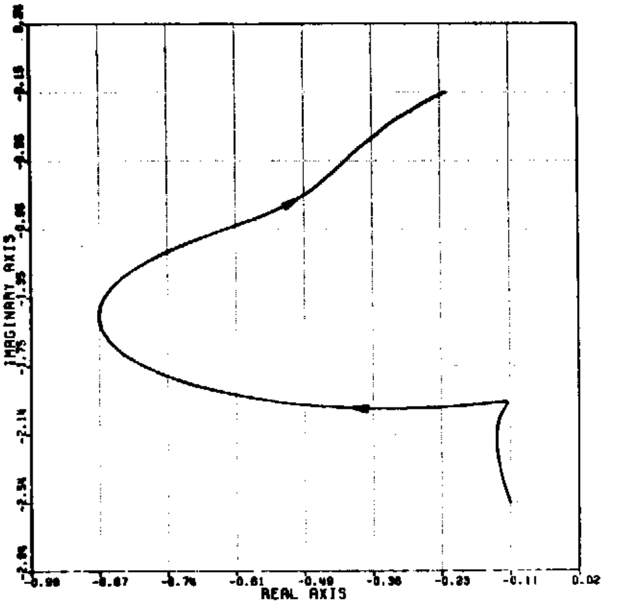

Figure 6 :Nyquist Plot $\Delta E_T(s)/\Delta V_R(s)$ for Generator No. 8 - Stabilizers Added to Four Generators in the System

A frequently asked question concerns the detrimental effect stabilizers at other locations in the system may have on the gain margin of a particular generator stabilizer. There is a consensus that this effect is not very significant, but this remains to be quantified. The frequency response program described in this paper allows an easy determination of this effect.

Table 1 contains the results obtained for the same nine-machine system, which in the absence of stabilizers shows instability due to AVR negative damping at generator No. 9. The maximum gain defined in this table corresponds to the stabilizer gain which would cause the "exciter mode" of generator No. 9 to go unstable. The minimum gain is that value below which the stabilizer action would not prevent system instability caused by AVR negative damping.

The gain margin of the stabilizer at generator No. 9 was initially set to approximately one-third of the instability gain, as in normal practice [3]. Note that the gain margin is unaffected by the addition of stabilizers to the other machines. The minimum gain required to stabilize the system, as shown in Table 1, is initially 0.1. As other stabilizers are added to the system this gain is generally reduced, showing a damping contribution from the other stabilizers to this troublesome mode. In one occasion the addition of another stabilizer causes a small increase in the minimum gain, indicating an unstabilizing effect which is however of no significance compared to the normal settings of stabilizer gains. It is also seen from Table 1 that a stabilizer is not needed at generator No. 9

(minimum gain is zero) in order to maintain system stability as long as stabilizers are fitted to most of the other generators.

NUMBER OF STABILIZERS IN THE SYSTEM	CRITICAL VALUES OF STABILIZER GAIN FOR GENERATOR No. 9	
	MAXIMUM*	MINIMUM*
1	3.33	.108
2	3.33	.108
3	3.33	.111
4	3.31	.090
5	3.28	.046
6	3.29	.038
7	3.26	.026
8	3.23	0.0
9	3.23	0.0

Table 1 : Gain margins for stabilizer at generator No. 9 as affected by the addition of stabilizers to other generators

* These values are given as multiples of the actual stabilizer gain.

The results listed in Table 1 and those obtained from the analysis of other systems strongly indicate that widespread use of stabilizers in power systems is a very sound strategy.

7. Conclusions

A small-scale version of the proposed eigenvalue algorithm showed very promising results and a large scale version is presently under development. It is thought this algorithm will be useful in the analysis of low damped electromechanical oscillations in large power systems.

The frequency response technique described in this paper has proved valuable in locating the most effective points in the multimachine system for placing the damping effort and also in tuning power system stabilizers. This technique allows the evaluation of stabilizer gain margins as affected by the addition of power system stabilizers to other generators in the system. It is also an important instrument for verification of the adequacy of stabilizer tuning in the multimachine environment based on approximate field measurements of GEP(s) [3].

The results shown for the nine machine system and those obtained for other systems gave no evidence that interaction between the various stabilizers in the system can be detrimental to overall stability. The problems involving intra-plant modes as described in [17] must be regarded as a special case of detrimental stabilizer interaction. The results listed in Table 1 show that widespread use of stabilizers in the system may prevent system instability even on the occurrence of stabilizer failure at the most critical point in the system. The results of this work are favourable to the recommendation that stabilizers should be installed in all new generating plants, even when not needed for local mode damping since they can provide damping to inter-area modes under normal operation and during contingencies [3].

The frequency response studies described in this paper were made using a linear impedance representation for all system loads. Future studies will involve determining power system damping requirements in the presence of a large amount of non-linear loads. Aspects of coordinating generator stabilizers with other modulation signals in HVDC links and static compensators will also be investigated.

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