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Computing Dominant Poles of Power System Multivariable Transfer Functions

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Introduction (1/3)

- Recent developments and increased use of modal analysis in studies of electrical, mechanical and civil engineering as well as in many other fields
- Good opportunities for use of modal equivalents in power system dynamics and control, harmonic analysis and real-time simulations of electromagnetic transients
- The concept of transfer function pole dominance has been little exploited in Numerical Linear Algebra
- First power system applications: AESOPS algorithm [Byerly, 1978; Kundur, 1990] and Selective Modal Analysis [Pagola, 1988]

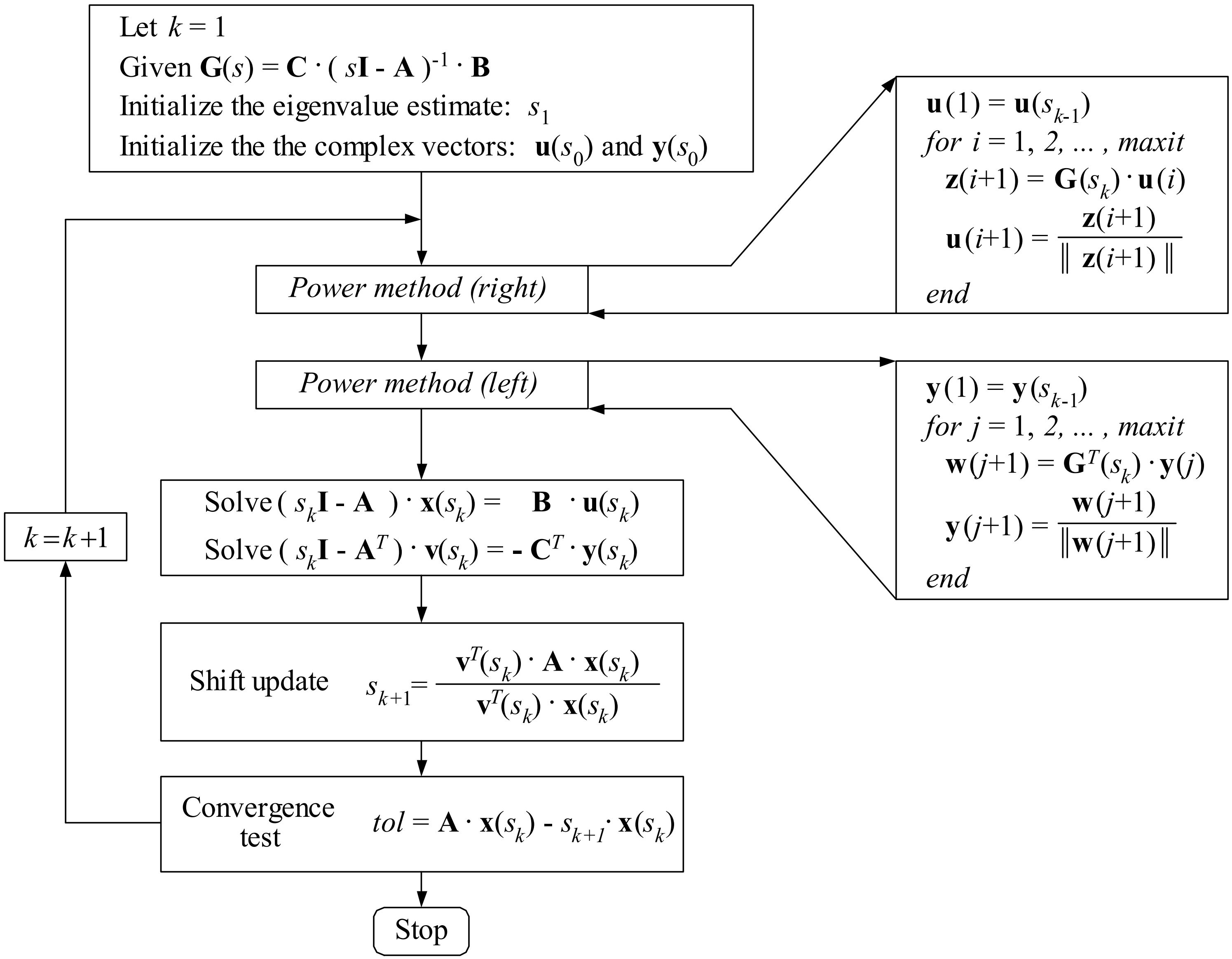
Introduction (2/3)

- Need for improved numerical robustness and more general eigensolution selectivity in small-signal stability analysis and power system controller design
- Clever implementation of Newton- Raphson algorithm applied to specified transfer functions: the Dominant Pole Algorithm (DPA) described in [Martins et alli, 1996]. Solves for one eigenvalue at a time.
- A generalization of DPA later described in [Martins, 1997], the Dominant Pole Spectrum Eigensolver (DPSE), can simultaneously solve for several dominant poles of a given scalar transfer function F(s)

Introduction (3/3)

- The Multivariable Dominant Pole algorithm (MDP), described in this paper, is a generalization of the DPA and solves for dominant poles of a given multivariable (MIMO) transfer function F(s)
- MDP is a clever implementation of the Newton-Raphson algorithm applied to a given matrix function F(s).
- MDP is a one-eigenvalue-at-a-time method, but makes efficient use of deflation techniques to find subdominant poles of F(s).

The MDP Algorithm



Basic Concepts Leading to Modal Equivalents of Scalar F(s)

Partial Fraction Expansion	$F(s) = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i}$
Step Input	$y(s) = F(s) \cdot \frac{1}{s} \approx \sum_{i=1}^{p} \frac{R_i}{s - \lambda_i} \cdot \frac{1}{s}$
Inverse Laplace Transform	$y(t) \approx \sum_{i=1}^{p} \frac{R_i}{\lambda_i} (e^{\lambda_i \cdot t} - 1)$

Modal Equivalents of Multivariable Transfer Functions

- An m x m transfer function $\mathbf{G}(s)$ may be expanded in terms of the system poles and associated residue matrices that also has dimension m x m:

$$\mathbf{G}(s) = \sum_{i=1}^{n} \frac{\mathbf{R}_i}{s - \lambda_i}$$

- The truncated sum below is the modal equivalent:

$$\mathbf{G}(s) \approx \sum_{i=1}^{p} \frac{\mathbf{R}_i}{s - \lambda_i} \text{, where } p << n$$

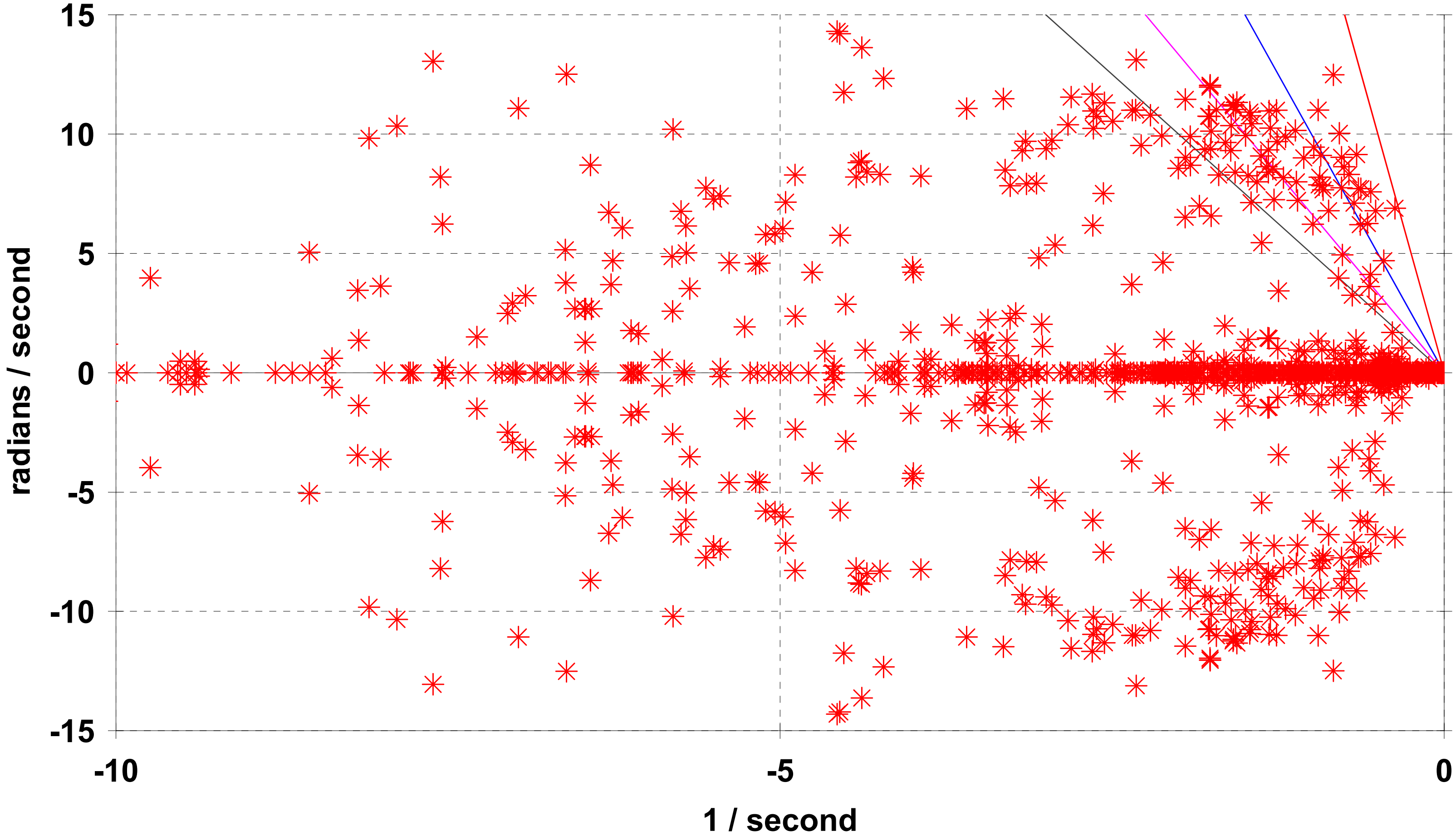
MDP Results on the North-South Brazilian System (Power System Operations Model for year 2,000)

- 2370-Bus, 3401 lines, 60,000 MVA generating capacity
- 123 machines, 99 speed-governors, 46 PSSs
- 4 SVCs, 2 TCSCs, one 6,000 MW HVDC link
- Descriptor system matrix of order 13,062 with 1,676 state variables
- Multivariable Transfer Function **G**(s) is a (8 x 8) matrix

Matrix Structure of the N-S Interconnection

Descriptor System Matrix has 13 K lines and 48 K nonzero elements

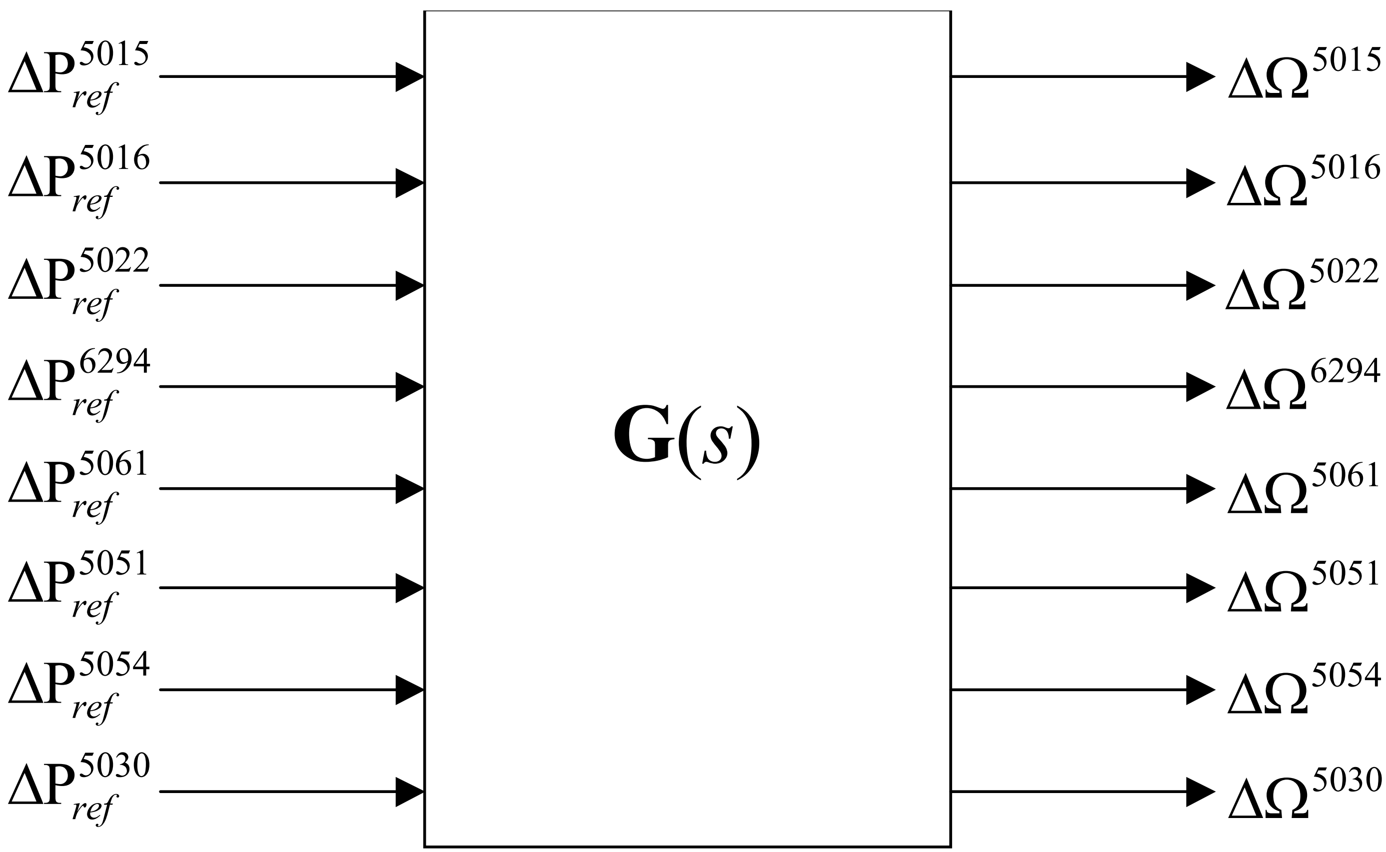
QR Eigensolution Results



Eigenvalue Spectrum of North-South Interconnection

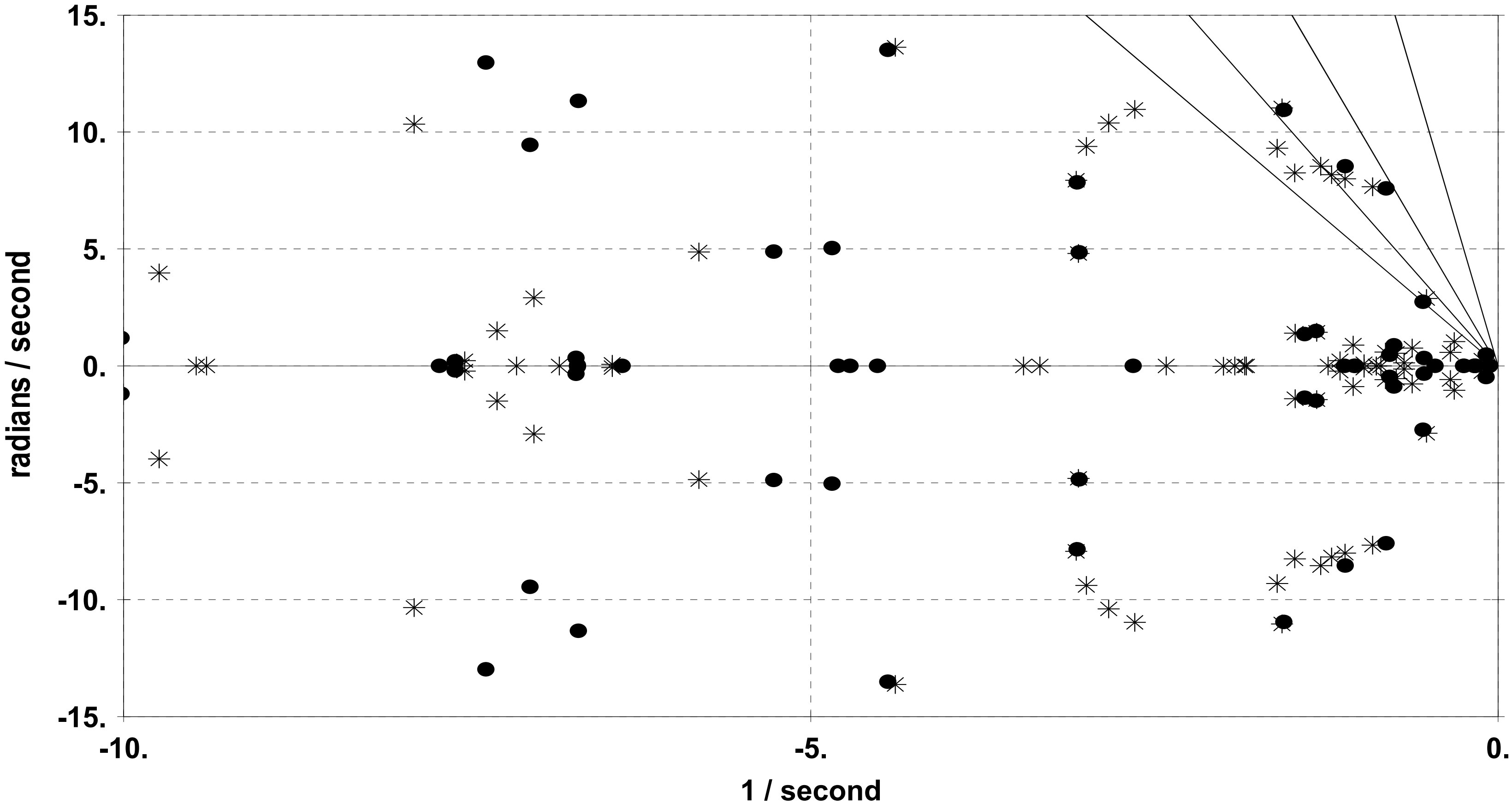
MDP Results for **G**(s)

Multivariable Transfer Function **G**(s) has Dimension (8 x 8)



Dominant Pole-Zero Spectrum of $\mathbf{G}(s)_{8 \times 8}$

- Full Model has order 1,676, but there is a large pole-zero cancellation



Poles pictured by asterisks and zeros by black circles

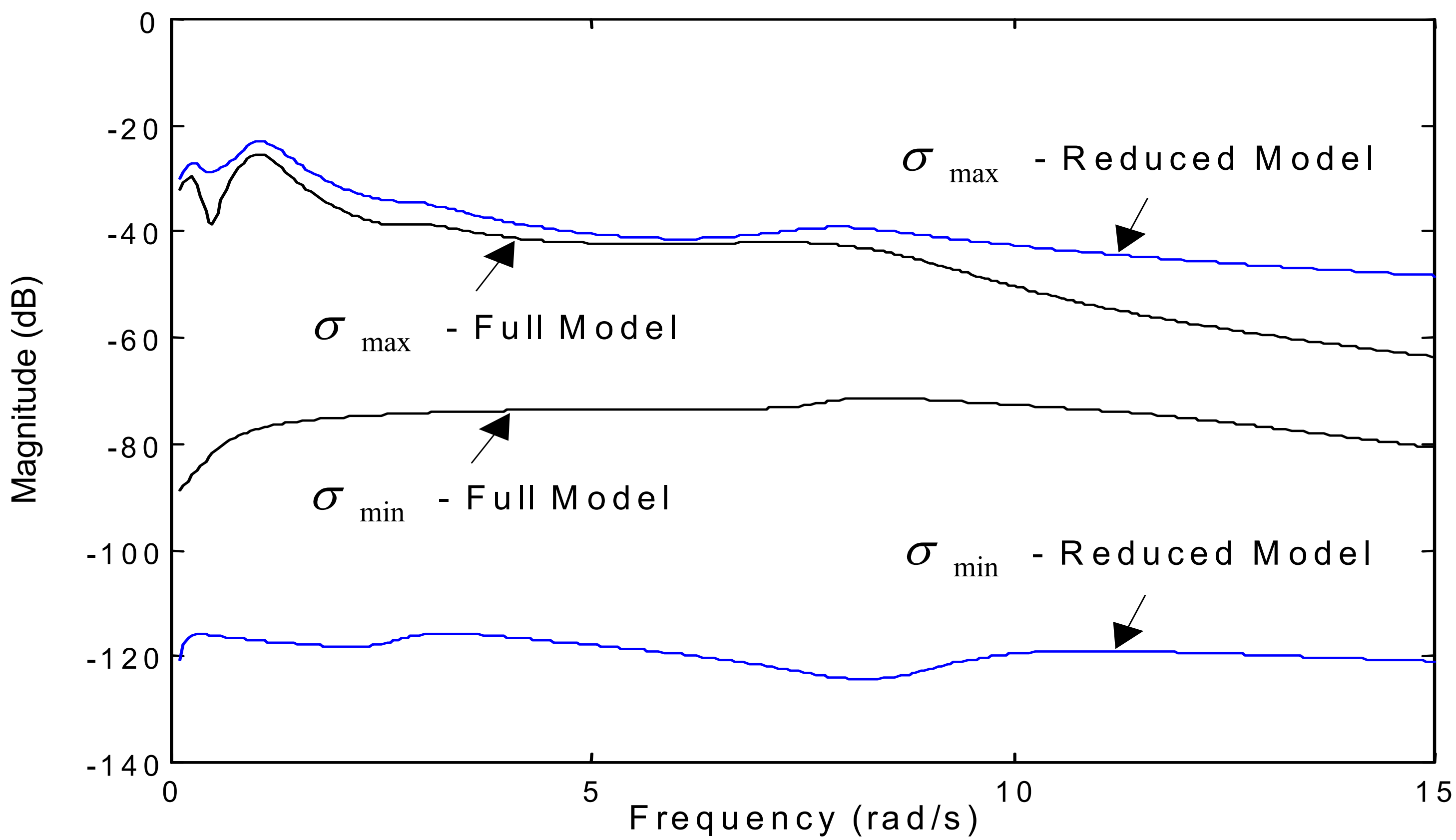
MDP Results for $\mathbf{G}(s)_{8 \times 8}$

Performance of the MDP Algorithm	
Initial Shift *s*	Converged Eigenvalues
–0.0759 +0.5j	–0.1158 +0.2445j (12)
–0.1517 +1.0j	–0.3179 +1.0437j (6)
–0.4551 +3.0j	–0.5199 +2.8814j (6)
–0.9103 +6.0j	–1.2098 +8.1765j (7)
–1.2137 +8.0j	–1.1129 +8.0075j (4)
–1.2896 +8.5j	–1.2902 +8.5407j (6)
–1.3654 +9.0j	–1.4778 +8.2550j (6)
–2.2757 +15.0j	–2.8323 +10.3949j (6)

The numbers within parenthesis denote iterations required for tight convergence (1.0e-10)

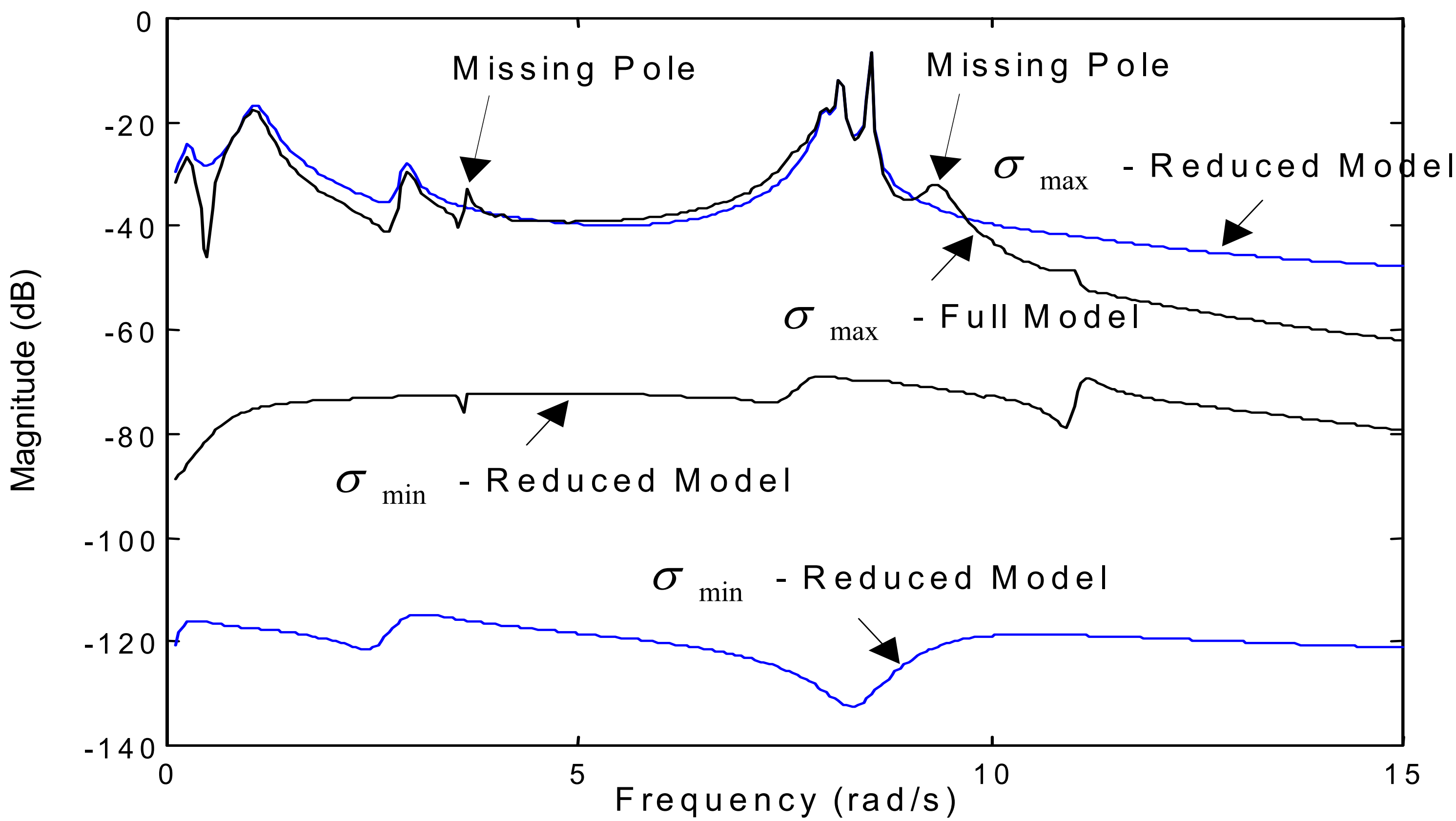
Modal Equivalent of T. Function **G**(s)$_{8 \times 8}$

- Sigma-plot for 8 x 8 **G**(*s*), $\xi = 0$
- Full Model order is 1,676. Modal Equivalent has order 16



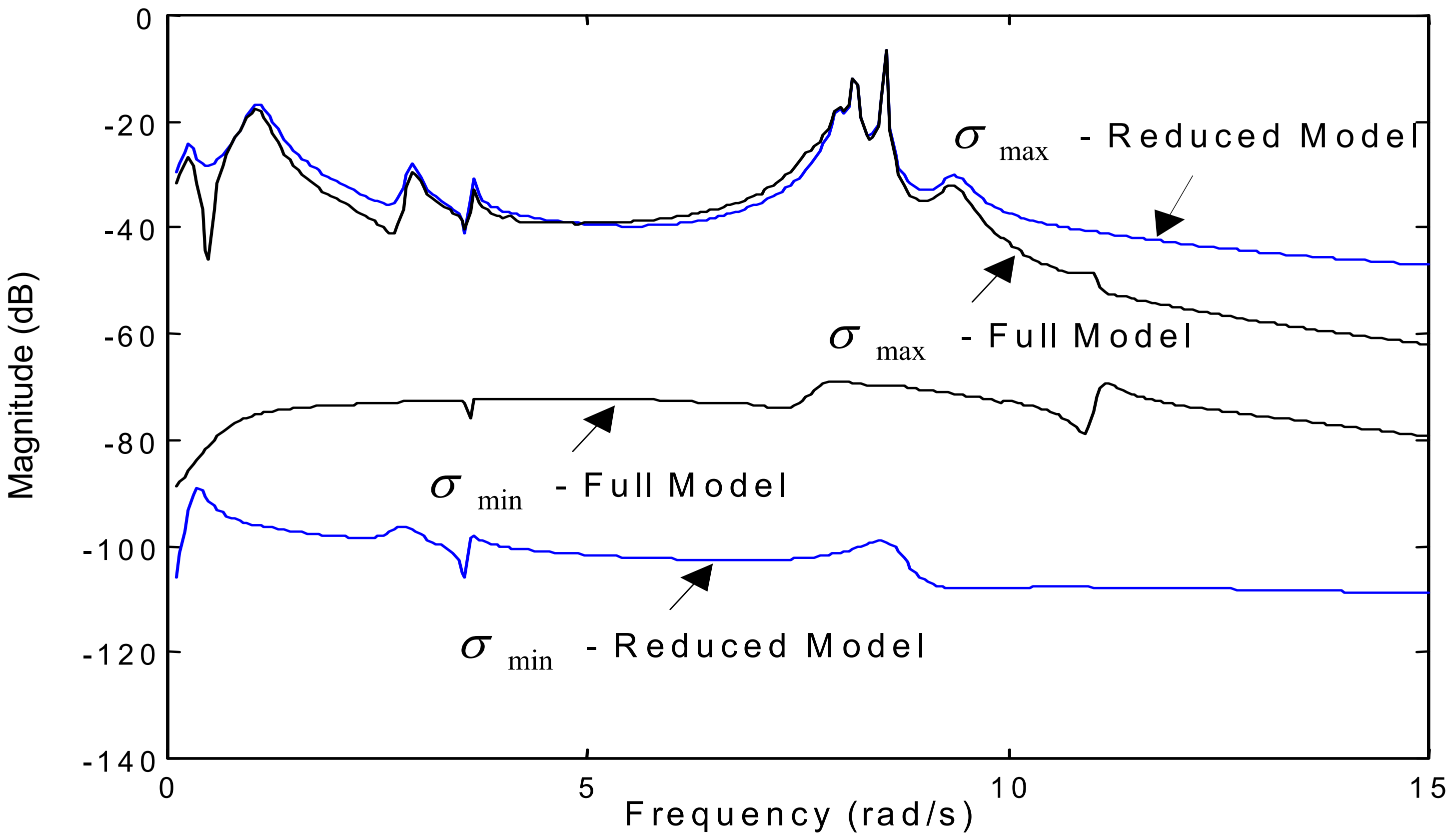
Modal Equivalent of T. Function **G**(s)$_{8 \times 8}$

- Sigma-plot for 8 x 8 **G**(*s*), $\xi = 15\%$
- Full Model order is 1,676. Modal Equivalent has order 16



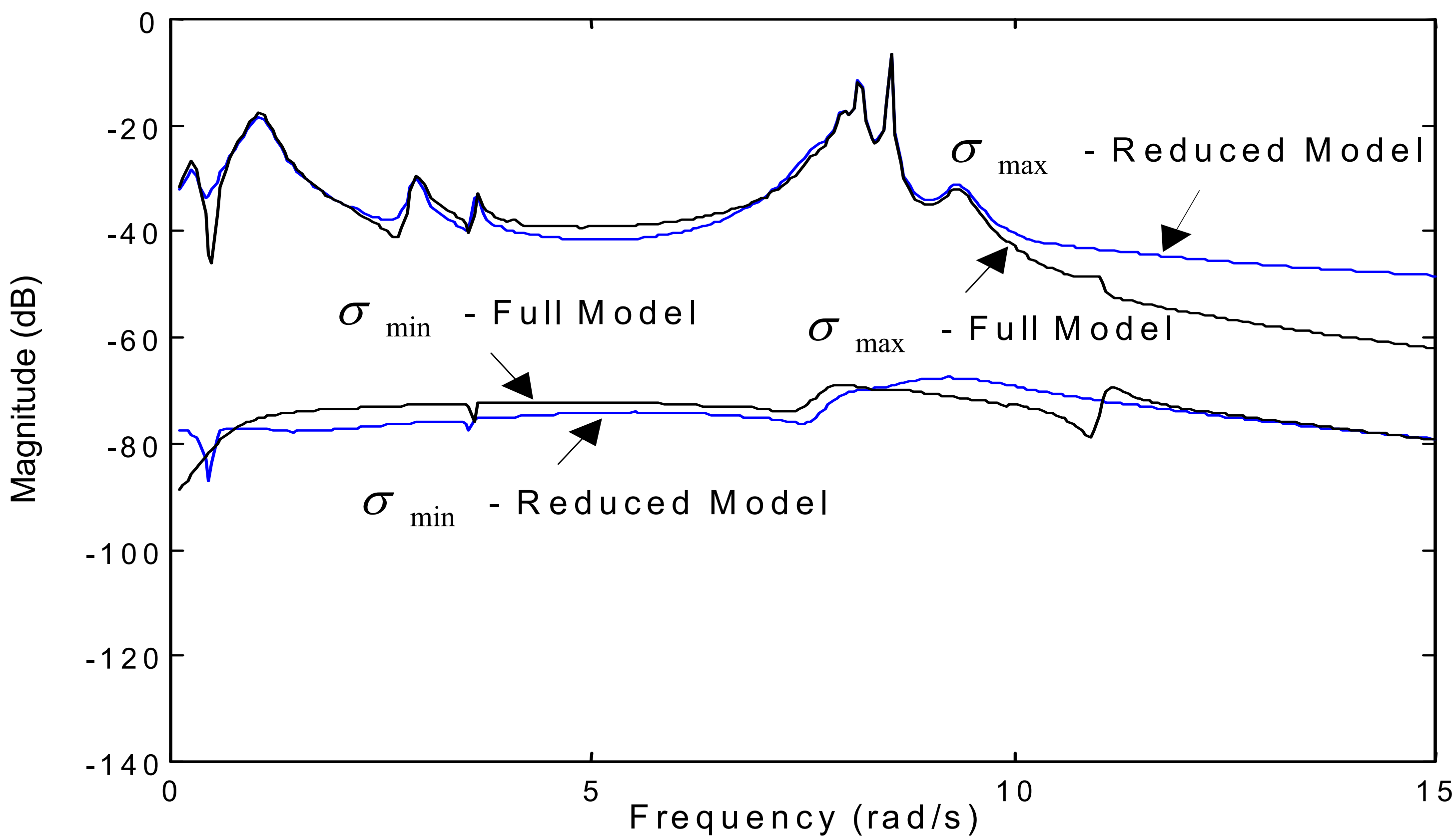
Modal Equivalent of T. Function **G**(s)$_{8 \times 8}$

- Sigma-plot for 8 x 8 **G**(*s*), $\xi = 15\%$
- Full Model order is 1,676. Modal Equivalent now has order 20, as the 2 missing complex poles have been included



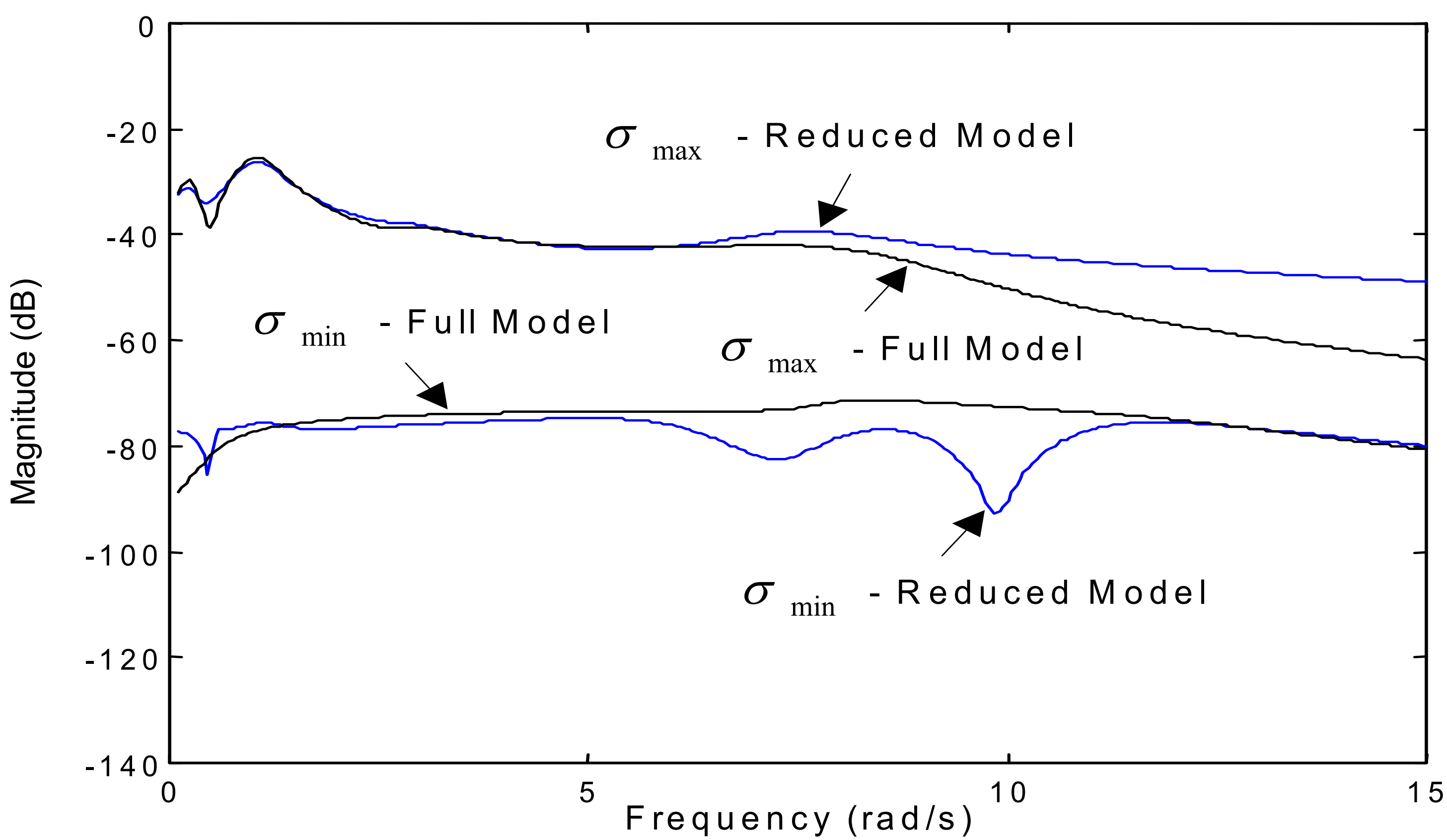
Modal Equivalent of T. Function **G**(s)$_{8 \times 8}$

- Sigma-plot for 8 **x** 8 **G**(*s*), $\xi = 15\%$
- Full Model order is 1,676. Modal Equivalent has order 39



Modal Equivalent of T. Function **G**(s)$_{8 \times 8}$

- Sigma-plot for 8 x 8 **G**(*s*), $\xi = 0$
- Full Model order is 1,676. Modal Equivalent has order 39



Step Responses $y_{ij}(s)$ for 39th - order Modal Equivalent for $g_{ij}(s)$

$$y_{ij}(t) \cong \frac{R_{ij}^{1}}{\lambda_1}(e^{\lambda_1 \cdot t} - 1) + \frac{R_{ij}^{2}}{\lambda_2}(e^{\lambda_2 \cdot t} - 1) +$$

$$+ \frac{R_{ij}^{3}}{\lambda_3}(e^{\lambda_3 \cdot t} - 1) + \cdots + \frac{R_{ij}^{39}}{\lambda_{39}}(e^{\lambda_{39} \cdot t} - 1)$$

where the complex conjugate poles and residues are imbedded in the above equation

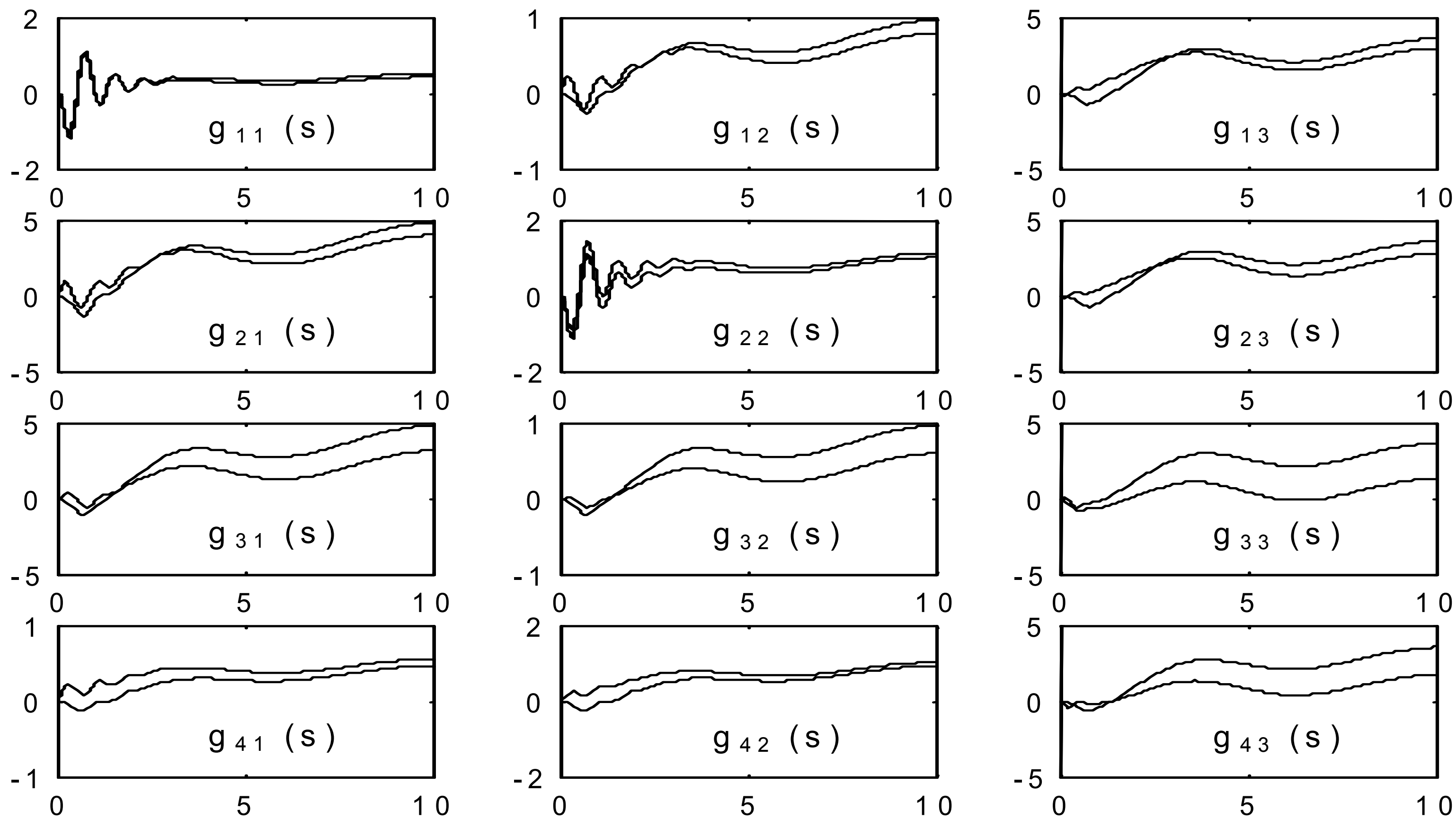
$$\lambda_1 = -.1158 + .2445j \qquad \lambda_2 = -.1158 - .2445j$$

$$\lambda_3 = -.3179 + 1.0437j \qquad \lambda_4 = -.3179 - 1.0437j$$

$$\lambda_5, \lambda_6, \cdots, \lambda_{38}, \lambda_{39}$$

Modal Equivalent of T. Function $\mathbf{G}(s)_{8x8}$

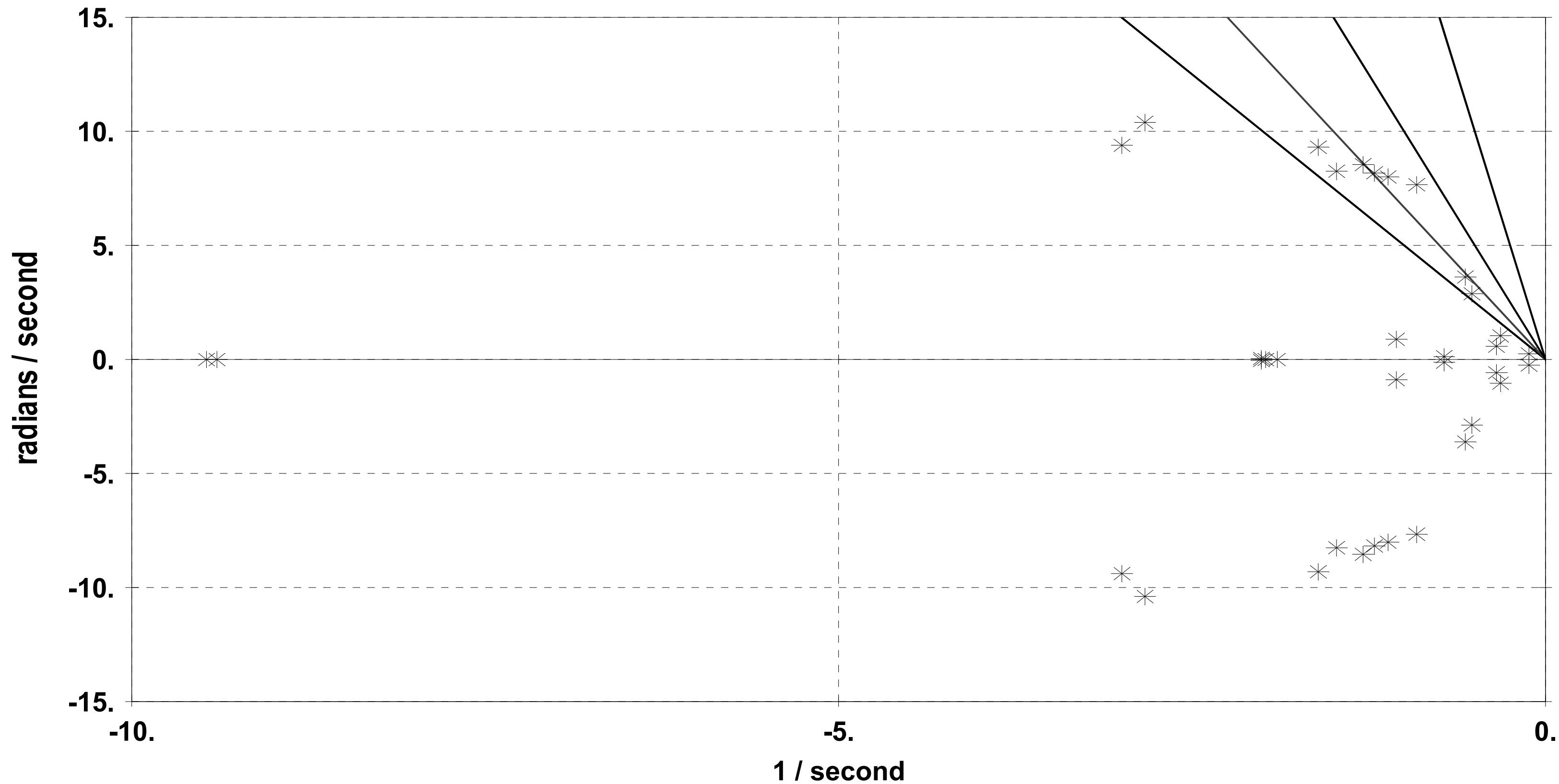
- Step responses for $g_{ij}(s)$ scalar transfer functions for the full model and the 39th-order modal equivalent



➢ **Note: Vertical axes given in rad/s and horizontal axes in seconds**

Dominant Poles of **G**(s)$_{8 \times 8}$

- Full model order is 1,676. Modal equivalent has order 39. All poles computed by the MDP algorithm



Only poles are pictured (by asterisks) in this figure

Conclusions

- The Multivariable Transfer Function Dominant Pole (MDP) algorithm operates on the state-space or the sparser descriptor system models of large dynamic systems
- MDP is a clever implementation of the Newton Raphson eigensolution algorithm applied to the multivariable transfer function: $\mathbf{G}(s) = \mathbf{C} \cdot (s\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$
- Convergence domain of the eigensolutions are larger for poles having high controllability/observability in $\mathbf{G}(s)$
- Subdominant poles of the multivariable $\mathbf{G}(s)$ are obtained by using other initial estimates and eigenvalue deflation techniques
- May automatically produce modal equivalents of $\mathbf{G}(s)$