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APPLYING SENSITIVITY ANALYSIS TO IMPROVE HARMONIC VOLTAGE PERFORMANCE

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Abstract – A method to approximately shift the electrical network poles and zeros to more suitable locations in the complex plane was proposed in [1]. These shifts are made in order to improve the harmonic voltage performance of a system. The method utilizes eigenvalue sensitivity coefficients that are efficiently obtained. The poles and/or zeros shifts are carried out by changes in the system elements (e.g. capacitor and/or reactor banks). As an improvement to the method described in [1], this paper presents a Newton-Raphson method to accurately carry out the required shifts. Although the mathematical formulation considers just one variable, it is possible to generalize the method to consider several variables and also some constraints e.g. maintenance of the system operating point. The main aspects of the method are described together with some results on a practical system problem.

Keywords: Harmonics, Newton-Raphson, Pole Shifts, Transfer Function Zeros, Eigenvalue Sensitivities.

1. Introduction

The harmonic voltage performance of a system depends on the location of its poles and zeros mainly with respect to the typical harmonic frequencies [2], [3]. Based on this fact, the harmonic voltage performance can be improved by shifting the electrical network poles and zeros to more suitable locations in the complex plane.

This paper presents a Newton-Raphson method that accurately accomplishes these shifts. Though not dealt with in this paper, it is possible to generalize the described method to simultaneously shift several zeros or poles, while imposing some important practical constraints, like maintaining the same operating point for the study system.

The methods present here and in [1] are based on the use of the poles and zeros sensitivities to changes in system parameters [1], [2], [3]. These sensitivities are efficiently obtained by modeling the network through the descriptor system approach [4], [5], [6] that has an inherent ability to deal with state variable redundancies (network degeneracy [7]). Additionally, it makes the pole and zero sensitivity calculations [1] suitable for practical application to large networks of any topology.

This paper presents a review of the descriptor system approach applied to the study of harmonic problems, a Newton-Raphson method to shift poles and zeros and example results on a practical system.

2. Network Modeling

The dynamic behavior of an electrical network is governed by: Kirchhoff's current law (KCL), Kirchhoff's voltage law (KVL) and the equations describing the inherent dynamic characteristics of each network element [8].

The Kirchhoff's laws (KCL and KVL) are algebraic equations containing the information on system topology. Each algebraic equation determines a linear dependence among system variables (voltages and currents). The dynamic characteristics of the inductive and capacitive elements are described by first-order differential equations, in terms of currents and voltages. The inductive currents and capacitive voltages represent the obvious choice of state variables.

However, building a dynamic model for a practical electrical network, based on the conventional state-space methodology may not be a simple task. By definition, the states form a minimum set of variables able to describe the dynamic behavior of a system [9]. Therefore, a minimum set of inductive currents and capacitive voltages, which are linearly independent, must be determined. The available techniques to determine this minimum set of states involve an elaborate topological analysis of the electrical circuit [7].

This difficulty is, however, overcome when using the descriptor system (or partially dynamic system) to model the electrical network [4], [5], [6]. The network modeling by the descriptor system technique assumes that all the inductive currents and all capacitive voltages are state variables. The algebraic constraints imposed by the KCL are also included in the model.

3. Single-phase RLC Series Branch

Harmonic studies usually utilize positive-sequence network models [10], [11], where only single-phase representation is necessary. Three-phase modeling, needed in some harmonic studies [11], [12], [13] could also be considered by the proposed method [5].

The single-phase RLC branch depicted in Fig. 1 is assumed to be the elementary network component.

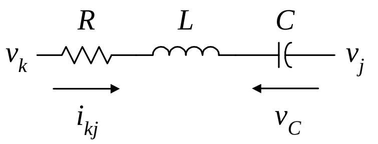

Fig. 1: RLC branch

The dynamic behavior of the RLC branch can be described by a set of two differential equations:

$$v_k - v_j = R\, i_{kj} + L \frac{di_{kj}}{dt} + v_C \qquad (1)$$

$$C \frac{dv_C}{dt} = i_{kj} \qquad (2)$$

Equation (1) is general and holds for the particular cases where L or R are zero. However, when there is no capacitor in the branch, (2) must be replaced by:

$$v_C = 0 \qquad (3)$$

4. Descriptor System for Single-phase RLC Networks

A given network can be represented for harmonic studies by the interconnection of single-phase RLC branches. For each element, (1) and (2) can be written in matrix form:

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{kj} \\ v_C \end{bmatrix} = \begin{bmatrix} -R & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_{kj} \\ v_C \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_k + \begin{bmatrix} -1 \\ 0 \end{bmatrix} v_j \qquad (4)$$

where the current i_{kj} through the inductor and the voltage v_C across the capacitor are the chosen state variables. Symbols v_k and v_j denote the voltages at nodes k and j, respectively.

If there is no capacitor, (4) needs to be modified:

$$\begin{bmatrix} L & 0 \\ 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{kj} \\ v_C \end{bmatrix} = \begin{bmatrix} -R & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{kj} \\ v_C \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_k + \begin{bmatrix} -1 \\ 0 \end{bmatrix} v_j \qquad (5)$$

Note that i_{kj} in (4) is positive if the current flows from node k to node j, i.e., a positive current injection at j and negative at k. The interconnection among the various network elements is modeled by a set of equations describing the KCL applied to each system node: the algebraic sum of all branch currents leaving a node is zero at all instants of time. Therefore, the current i_{kj} is assumed positive in the equation for the currents associated with the *j-th* node and assumed negative in the equation associated with the *k-th* node. If a given branch is connected to ground $(j = 0 \text{ or } k = 0)$, the current through this branch will only be present in one equation.

The electric network model contains two differential equations for each existing RLC branch and one algebraic equation (the KCL) per system node. After interconnecting the equations for all RLC branches, the following descriptor system equation is obtained:

$$\left[\begin{array}{c|c} \mathbf{T}_1 & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_q \end{array}\right] \left[\begin{array}{c} \dot{\mathbf{x}}_1 \\ \hline \dot{\mathbf{v}}_{nodal} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{A}_1 & \mathbf{A}_2 \\ \hline \mathbf{A}_3 & \mathbf{0}_q \end{array}\right] \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{array}\right] + \left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{I} \end{array}\right] \mathbf{i}_{nodal} \qquad (6)$$

$$\mathbf{v}_{nodal} = \left[\mathbf{0}^T \;\middle|\; \mathbf{I}\right] \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{array}\right] \qquad (7)$$

$\mathbf{T}_1$ is a diagonal matrix and $\mathbf{A}_1$ block-diagonal; $\mathbf{A}_2$ and $\mathbf{A}_3$ are "incidence matrices" for the descriptor system. Symbols $\mathbf{0}$ and $\mathbf{0}_q$ denote null matrices and $\mathbf{I}$ is the identity matrix. The superscript ^{T} denotes matrix transposition and a dot over a vector its time derivative.

The matrix equations (6) and (7) can be written in compact form:

$$\mathbf{T}\,\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \qquad (8)$$

$$\mathbf{y} = \mathbf{C}\,\mathbf{x} \qquad (9)$$

where:

$$\mathbf{T} = \left[\begin{array}{c|c} \mathbf{T}_1 & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_q \end{array}\right] \qquad (10) \qquad \mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_1 & \mathbf{A}_2 \\ \hline \mathbf{A}_3 & \mathbf{0}_q \end{array}\right] \qquad (11)$$

$$\mathbf{B} = \left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{I} \end{array}\right] \qquad (12) \qquad \mathbf{C} = \left[\mathbf{0}^T \;\middle|\; \mathbf{I}\right] \qquad (13)$$

$$\mathbf{u} = \mathbf{i}_{nodal} \qquad (14) \qquad \mathbf{x} = \left[\begin{array}{c} \mathbf{x}_1 \\ \hline \mathbf{v}_{nodal} \end{array}\right] \qquad (15)$$

$$\mathbf{y} = \mathbf{v}_{nodal} \qquad (16)$$

Let n_l be the number of RLC branches and n_n the number of network nodes. The various matrix and vector dimensions are listed in Table 1.

Table 1: Matrix and vector dimensions

	Symbol	Dimensions
Matrices	$\mathbf{A}_1$, $\mathbf{T}_1$	$2n_l \times 2n_l$
	$\mathbf{A}_2$, $\mathbf{0}$	$2n_l \times n_n$
	$\mathbf{A}_3$	$n_n \times 2n_l$
	$\mathbf{I}$, $\mathbf{0}_q$	$n_n \times n_n$
	$\mathbf{A}$, $\mathbf{T}$	$(2n_l + n_n)\times(2n_l + n_n)$
	$\mathbf{B}$	$(2n_l + n_n)\times n_n$
	$\mathbf{C}$	$n_n \times (2n_l + n_n)$
Vectors	$\mathbf{x}_1$	$2n_l$
	$\mathbf{v}_{nodal}$, $\mathbf{i}_{nodal}$, $\mathbf{u}$, $\mathbf{y}$	n_n
	$\mathbf{x}$	$2n_l + n_n$

5. Harmonic Impedance seen from a System Node

Applying the Laplace Transform to (8) and (9):

$$\mathbf{x}(s) = (s\,\mathbf{T} - \mathbf{A})^{-1}\,\mathbf{B}\,\mathbf{u}(s) \tag{17}$$

$$\mathbf{y}(s) = \mathbf{C}\,\mathbf{x}(s) \tag{18}$$

where $\mathbf{x}(s)$, $\mathbf{u}(s)$ and $\mathbf{y}(s)$ are the Laplace transforms of **x**, **u** and **y**, respectively.

From the above two equations:

$$\mathbf{y}(s) = \mathbf{C}\,(s\,\mathbf{T} - \mathbf{A})^{-1}\,\mathbf{B}\,\mathbf{u}(s) \tag{19}$$

The impedance matrix $\mathbf{Z}(s)$ can be defined from (19):

$$\mathbf{Z}(s) = \mathbf{C}\,(s\,\mathbf{T} - \mathbf{A})^{-1}\,\mathbf{B} \tag{20}$$

The self-impedance seen from node *k* is given by the z_{kk} element of the $\mathbf{Z}(s)$ matrix. However, from (20) and (10) to (13), one concludes that the z_{kk} element is equal to the $(2n_l + k)$ diagonal element of $(s\,\mathbf{T} - \mathbf{A})^{-1}$. Thus:

$$z_{kk}(s) = \mathrm{diag}\left[(s\,\mathbf{T} - \mathbf{A})^{-1}\right]_{(2n_l + k)} \tag{21}$$

The inverse of $(s\,\mathbf{T} - \mathbf{A})$ is given by:

$$(s\,\mathbf{T} - \mathbf{A})^{-1} = \frac{\mathrm{adj}(s\,\mathbf{T} - \mathbf{A})}{\det(s\,\mathbf{T} - \mathbf{A})} \tag{22}$$

Let $\mathbf{T}_k$ and $\mathbf{A}_k$ be the matrices obtained by canceling the $2n_l + k$ row and column of the matrices **T** and **A**, respectively. Thus, the $2n_l + k$ diagonal element of $(s\,\mathbf{T} - \mathbf{A})^{-1}$ is given by:

$$z_{kk}(s) = \mathrm{diag}\left[(s\,\mathbf{T} - \mathbf{A})^{-1}\right]_{(2n_l + k)} = \frac{\det(s\,\mathbf{T}_k - \mathbf{A}_k)}{\det(s\,\mathbf{T} - \mathbf{A})} \tag{23}$$

Equation (23) applies for descriptor systems, being a generalization of [2], [3] developed for conventional state space systems. It shows that:

The system poles are the generalized eigenvalues [14] of the matrix pair $\{\mathbf{A}, \mathbf{T}\}$:

$$\det(s\,\mathbf{T} - \mathbf{A}) = 0 \Leftrightarrow \mathbf{A}\,\mathbf{v}_i = \lambda_i\,\mathbf{T}\,\mathbf{v}_i \tag{24}$$

The zeros, associated with the self-impedance of node *k*, are the generalized eigenvalues of the matrix pair $\{\mathbf{A}_k, \mathbf{T}_k\}$:

$$\det(s\,\mathbf{T}_k - \mathbf{A}_k) = 0 \Leftrightarrow \mathbf{A}_k\,\mathbf{v}'_i = \lambda'_i\,\mathbf{T}_k\,\mathbf{v}'_i \tag{25}$$

where λ_i and λ'_i are the generalized eigenvalues associated with the pairs $\{\mathbf{A}, \mathbf{T}\}$ and $\{\mathbf{A}_k, \mathbf{T}_k\}$ and $\mathbf{v}_i$ and $\mathbf{v}'_i$ are their associated generalized eigenvectors.

6. Test System

The test system is shown in Fig. 2, and has previously been utilized in [3]. This system can be modeled by the interconnection of several series RLC branches, as shown in Fig. 3.

The system frequency is 50 Hz and the values of its elements are given in Table 2. L_{12} and R_{12} represent the inductance and resistance of the equivalent series association of the line *LT 1-2* with the transformer *T2*. Similarly, L_{13} and R_{13} represent the inductance and resistance of the equivalent series association of the line *LT 1-3* with the transformer *T3*. The impedance loads Z_2 and Z_3 are modeled as shunt reactors (L_2 and L_3) in parallel with shunt resistors (R_2 and R_3).

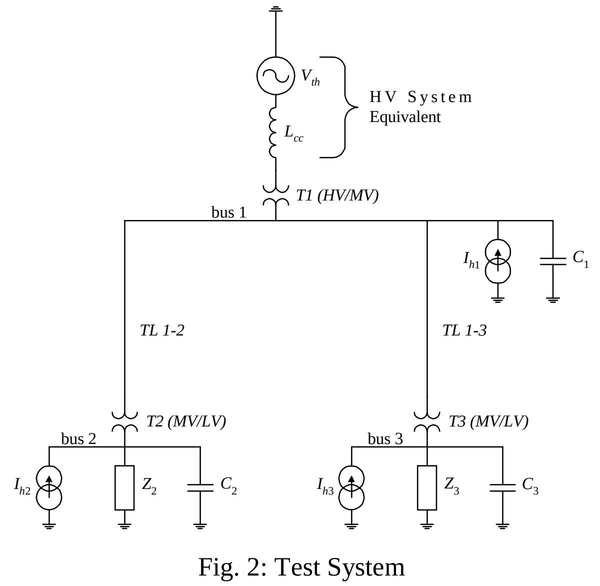

Fig. 2: Test System

V_{th}	:	Thévenin voltage.
L_{cc}	:	Short-circuit inductance of the HV system.
T1	:	HV/MV transformer.
T2, *T3*	:	MV/LV Transformers.
LT 1-2	:	Transmission line connecting bus 1 to transformer *T2*.
LT 1-3	:	Transmission line connecting bus 2 to transformer *T3*.
C_1, C_2, C_3	:	Capacitor banks connected to buses 1, 2 and 3, respectively.
Z_2, Z_3	:	Load impedances connected to buses 2 and 3, respectively.
I_{h1}, I_{h2}, I_{h3}	:	Harmonic current sources connected to buses 1, 2 and 3, respectively.

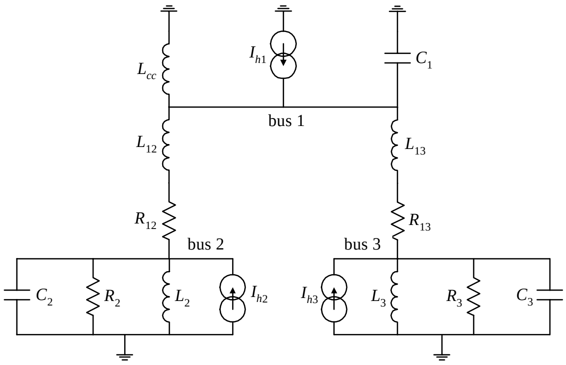

Fig. 3: System modeling

Table 2: System parameter values

Inductance (mH)		Resistance (Ω)		Capacitance (μF)	
L_{cc}	8.0	R_2	80.0	C_1	23.9
L_2	424.0	R_3	133.0	C_2	8.0
L_3	531.0	R_{12}	0.46	C_3	11.9
L_{12}	9.7	R_{13}	0.55		
L_{13}	11.9				

For the test system, the order of matrices **A** and **T** is 23. The sparse structure of the generalized state matrix (**A**) is depicted in Fig. 4.

The constant nz represents the number of nonzero elements, and is shown at the bottom of Fig. 4.

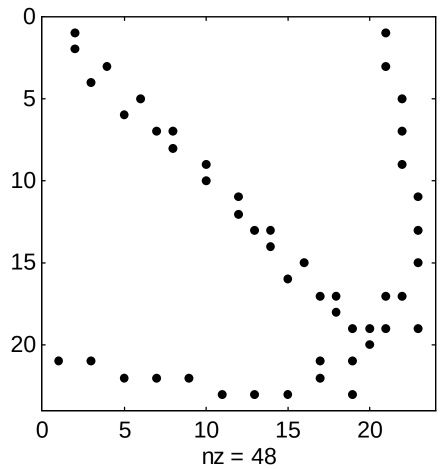

Fig. 4: Sparse structure of matrix **A** for the Test System

6.1 *Calculation of Poles, Zeros and Sensitivities*

Although the order of the matrices **A** and **T** is 23, the test system is actually of eighth-order. Therefore, 15 generalized eigenvalues of infinite moduli corresponding to the algebraic equations are obtained.

Matrices $\mathbf{A}_k$ and $\mathbf{T}_k$ have order 22. They also have 15 generalized eigenvalues of infinite moduli and 7 of finite module. The values of the finite generalized eigenvalues associated to matrices $\{\mathbf{A},\mathbf{T}\}$ (poles) and to matrices $\{\mathbf{A}_k,\mathbf{T}_k, k=1,2,3\}$ (zeros), are shown in Table 3.

Table 3: Generalized eigenvalues

$\{\mathbf{A},\mathbf{T}\}$	$\{\mathbf{A}_1,\mathbf{T}_1\}$	$\{\mathbf{A}_2,\mathbf{T}_2\}$	$\{\mathbf{A}_3,\mathbf{T}_3\}$
-345.9 $\pm j4535.6$	-338.5 $\pm j2670.9$	-93.7 $\pm j3975.6$	-398.4 $\pm j4424.9$
-507.0 $\pm j3069.1$	-804.4 $\pm j3550.6$	-255.5 $\pm j2084.9$	-415.3 $\pm j2402.1$
-290.1 $\pm j1583.6$	-1.0	-26.2	-27.8
-1.0	-1.1	-1.0	-1.0
-1.0	0.0	0.0	0.0

The frequencies (imaginary parts divided by 2π) of the complex conjugate network poles (parallel resonance) and zeros of the self-impedances of the three buses (series resonance), as well as their sensitivities (see [1], [2] and [3]) with respect to the inductances and capacitances of the Test System are presented in Table 4. The sensitivities are normalized, being given in *Hz*/per unit change of the nominal parameter value.

Table 4: Resonance frequencies and sensitivities

	System poles			Zeros seen from					
				Bus 1		Bus 2		Bus 3	
	1	2	3	1	2	1	2	1	2
f(Hz)	252	489	722	425	565	332	633	382	704
L_{cc}	-101	-11	-50	0	0	-48	-80	-88	-47
L_2	-3	-4	-2	0	-7	0	0	-5	-2
L_3	-4	-2	0	-5	0	-5	-1	0	0
L_{12}	-2	-78	-247	0	-290	-40	-66	-37	-289
L_{13}	-19	-151	-78	-211	0	-75	-172	-59	-32
C_1	-45	-12	-206	0	0	-38	-242	-85	-189
C_2	-25	-116	-111	0	-269	0	0	-109	-145
C_3	-54	-114	-28	-210	0	**-127**	-72	0	0

7. A Newton-Raphson Method for Shifting Poles and Zeros

7.1 *Mathematical Formulation*

Let f be the frequency value in *Hz* (imaginary part divided by 2π) of a chosen pole or zero of a system; f_r the target value for f (in other words, f should become equal to f_r at the solution) and p the value of a system parameter that is assumed to vary. One may define:

$$f = f(p) \tag{26}$$

The mismatch function of the frequency of the selected pole or zero can be defined by:

$$g(p) = \left(\frac{f(p) - f_r}{f_r}\right) 100\% \tag{27}$$

This equation must be satisfied at the solution:

$$g = 0 \tag{28}$$

Applying the Newton-Raphson method to (28), one obtains the following recurrence formula

$$p^{k+1} = p^k - \left[\left(\frac{dg}{dp} \right)^k \right]^{-1} g^k \qquad (29)$$

where the index k denotes the iteration number. The parameter estimate p^{k+1} tends to the solution p_r, at convergence.

The derivative of g with respect to p is given by:

$$\frac{dg(p)}{dp} = \frac{100}{f_r} \frac{df(p)}{dp} \qquad (30)$$

The values of $\frac{df(p)}{dp}$ correspond to the imaginary part (divided by 2π) of the eigenvalue sensitivity, directly obtained from (see [1]):

$$\frac{d\lambda}{dp} = \frac{1}{k} \mathbf{w} \left(\frac{d\mathbf{A}}{dp} - \lambda \frac{d\mathbf{T}}{dp} \right) \mathbf{v} \qquad (31)$$

where **v** and **w** are the *right (column)* and the *left (row)* generalized eigenvectors of $\{\mathbf{A}, \mathbf{T}\}$ associated with the generalized eigenvalue $\lambda = \sigma + j\, 2\pi f$, and $k = \mathbf{w}\,\mathbf{T}\,\mathbf{v}$.

7.2 Results

As shown in Table 4, the pole located at 252 Hz is denoted by pole 1. It can cause problems at any system bus since an injection of fifth-harmonic current can generate high levels of harmonic distortions. Consider that a 5^{th} harmonic source exists in bus 2, and that harmonic voltage distortion is excessive.

A possible solution to this problem consists in bringing the zero 1 seen from bus 2 to the frequency of 250 Hz.

The highlighted column of sensitivity results in Table 4 indicate that changes in parameter C_3 will cause the largest shifts in the chosen zero.

Applying the Newton-Raphson algorithm in (29), the solution to this problem ($C_3 = 23.38\ \mu F$) was obtained in 4 iterations with an absolute mismatch function value less than 0.1 %. Further iterations reveal the method possesses quadratic convergence, which is typical of the Newton-Raphson method.

The impedance moduli seen from bus 2 associated with the original and the new value of C_3 are shown in Fig. 5. This new value of C_3 causes a reduction of 70% in the impedance magnitude at 250 Hz, yielding much less voltage distortion.

The poles and zeros loci, as seen from bus 2, as the parameter C_3 varies from 11.9 to 23.38 μF is depicted in Fig. 6. Note that only the poles and zeros with non-zero imaginary parts were plotted in this figure.

The frequencies of the poles and zeros, for the converged value of C_3, are presented in Table 5. Note that the poles 1 and 2 experienced the largest shifts following the change in parameter C_3, a result that could be anticipated from the sensitivities shown in Table 4. Fig. 5 shows that there is an impedance magnitude rise around the frequency of pole 2, since its damping factor reduces as C_3 is varied. The damping reduction experienced by pole 1 was compensated by an approximately similar shift of zero 1, so that the impedance seen from bus 2 actually shows a smaller magnitude at about 200 *Hz*. These conclusions are supported by the root locus plot (for poles and zeros) shown in Fig. 6.

Table 5: New frequencies of poles and zeros

	Poles			Zeros					
				Bus 1		Bus 2		Bus 3	
	1	2	3	1	2	1	2	1	2
f(Hz)	208	431	711	304	565	250	601	382	704

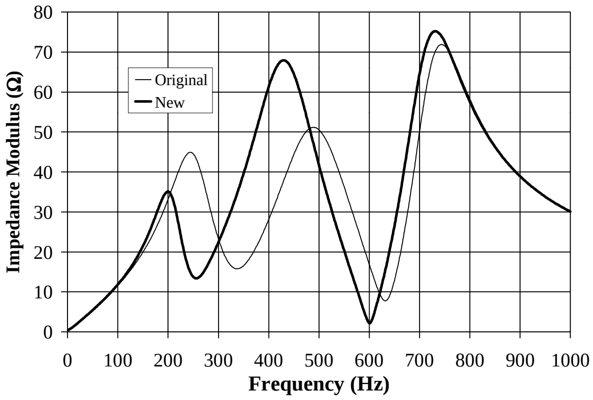

Fig. 5: Impedance modulus seen from bus 2

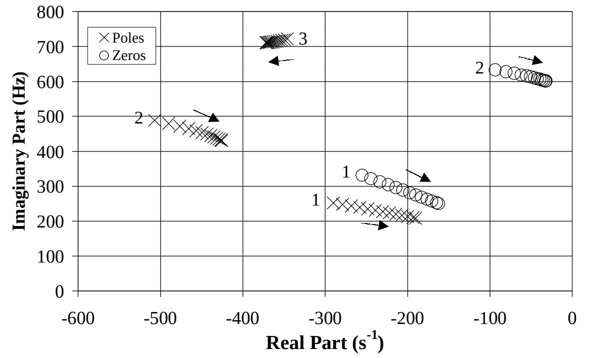

Fig. 6: Root locus for poles and zeros when varying C_3

Note that the numbers appearing beside the several root-locus branches in Fig. 6 identify the system poles and the zeros seen from bus 2, and are in accordance with tables 4 and 5.

8. Conclusion

The traditional method for harmonic analysis is based on nodal admittance matrices computed at various discrete values of frequency within the range of interest.

The more recent state space [2], [3] and descriptor system [1], [5], [6] methods allow obtaining all the results produced by the traditional method for linear models. Furthermore, these methods allow:

- Identification of the elements mostly associated with specific resonances.

- Determination of the necessary changes in system elements in order to shift the location of poles and/or zeros to desired positions.
- Optimum allocation of capacitor banks and/or passive filters.

The conventional state space method [2], [3] has, however, some limitations regarding its ability to model practical networks. The descriptor system method [1], [5], [6] overcomes these limitations and has others advantages:

- Simple and efficient computational implementation.
- Ability to model systems of any topology and containing state variable redundancies.
- Applicability to large-scale networks, due to the very sparse matrices involved and the availability of powerful sparse eigensolution algorithms applied to descriptor systems [5], [15], [16].

This paper describes, for the first time, a Newton-Raphson method that accurately shifts poles and zeros to more suitable locations, by varying inductances / capacitances in system reactive compensation banks or passive harmonic filters.

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