DECENTRALIZED DESIGN OF POWER SYSTEM DAMPING CONTROLLERS USING A LINEAR MATRIX INEQUALITY ALGORITHM

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Abstract— **This paper presents a systematic procedure based on Linear Matrix Inequalities (LMI) for the simultaneous tuning of multiple power system damping controllers. The method directly solves for a low-order controller which satisfies a frequency-domain robustness bound. The design formulation involves a nonconvex optimization problem, which is solved iteratively considering a set of linear matrix inequalities as design constraints. Controller constraints such as decentralization and positive realness (all zeros in the left half-plane) can also be included in the design. The paper illustrates the method with an application to the design of two damping controllers for a small-scale system which keeps some dynamic characteristics of the South-Southeast Brazilian System.**

Keywords— **Linear Matrix Inequalities, Decentralized Control, Robust Control, Small-Signal Stability, Convex Optimization.**

I. Introduction

THE purpose of this paper is to demonstrate the use of an iterative LMI-based optimization method to simultaneously design multiple low-order power system damping controllers. The method will be applied to the design of multiple power system stabilizers in a 7-bus equivalent of the power system model used in the initial planning studies of the Itaipu generation and AC transmission complex. The system shows two lightly damped inter-area modes that cannot be simultaneously controlled with a single PSS. This is a situation where controller coordination to avoid adverse interactions must be carefully analyzed.

The method utilized in this paper belongs to a class of H_∞ optimization techniques. Recently, several papers have discussed the use of H_∞ techniques to design robust power swing damping controllers [1], [2], [3]. It is shown that if the design is formulated as a mixed sensitivity or a model-matching problem using appropriate frequency weighting functions, the controller can be readily solved using available design packages. However, in the standard H_∞ problem, a designer cannot directly impose constraints on the controller. For example, the order of the controller has to be equal to the system order plus the order of the weighting functions, and thus it may be much higher than desired or needed. The controller may also have lightly damped poles and may even be non-minimum phase. In applications involving several control channels, as in power systems, it may be desirable to use a decentralized control structure.

The design method used in this paper will allow the designer to fix the order of the controller and impose appropriate constraints on its structure. It requires the control designer to select the controller order and the controller poles, based on design considerations such as controller bandwidth. For design problems with controller structural constraints such as fixed-order controller, it is well known that the design equation involves a biaffine matrix inequality (BMI) which is a non-convex programming problem and cannot be solved in polynomial time. In the proposed method, the BMI problem arising from a controller parameterization is formulated as a dual design which is solved as recursive sets of LMIs. Although the method does not have global convergence properties, it converges quite well to local minima.

The proposed method has been applied to decentralized control design when the interactions between the controllers are low. In the 7-bus system of this paper, it is anticipated that the control design including strong interactions between the controllers, would be a more challenging design problem.

II. Low-Order H_∞ Controller Design

Consider the linear time-invariant generalized plant $G(s)$ with the state-space realization

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_{g1} w + B_{g2} u \\ z &= C_{g1} x_g + D_{g12} u \\ y_g &= C_{g2} x_g \end{aligned} \tag{1}$$

where x_g is the n-dimensional state variable vector, w is the m_1-dimensional disturbance and other external input vector, z is the p_1-dimensional controlled output vector, u is the m_2-dimensional controlled input vector, and y_g is the p_2-dimensional measured output vector.

The low-order H_∞ controller design problem is to find a control $u = K(s)y$, as shown in Figure 1, such that the closed-loop transfer function from w to z, denoted as $T_{zw}(s)$, is stable and its H_∞ norm satisfies

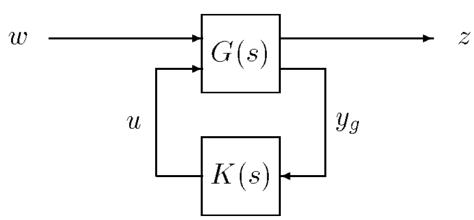

Fig. 1. Low-Order H_∞ Controller Design

$$\| T_{zw}(s) \|_\infty < \gamma \tag{2}$$

where $\gamma > 0$ is a pre-specified constant, and $K(s)$ has the state-space realization

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k y_g \\ u &= C_k x_k + D_k y_g \end{aligned} \tag{3}$$

where x_k is the n_c-dimensional control vector, and the controller order n_c is specified as $n_c < n$.

We will consider the special case of a low-order controller design in which the poles of the controller are already known. In many control systems, a designer may be able to specify the poles of the controller quite readily, usually based on the control bandwidth specifications. With the poles fixed, only the gains and the zeros of the low-order controller need to be optimized. In this case, we put the pair (A_k, B_k) in the controller canonical form ([8], p. 50), with the eigenvalues of A_k being the desired poles. Then the low-order controller design for the generalized system (1) can be reformulated as an H_∞ static output-feedback control problem of finding

$$u = \begin{bmatrix} C_k & D_k \end{bmatrix} y \tag{4}$$

for the generalized system

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x \end{aligned} \tag{5}$$

where

$$x = \begin{bmatrix} x_k \\ x_g \end{bmatrix}, \quad y = \begin{bmatrix} x_k \\ y_g \end{bmatrix}, \quad A = \begin{bmatrix} A_k & B_k C_{g2} \\ 0 & A_g \end{bmatrix} \tag{6}$$

$$B_1 = \begin{bmatrix} 0 \\ B_{g1} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ B_{g2} \end{bmatrix} \tag{7}$$

$$C_1 = \begin{bmatrix} 0 & C_{g1} \end{bmatrix}, \quad C_2 = \begin{bmatrix} I_{n_c} & 0 \\ 0 & C_{g2} \end{bmatrix}, \quad D_{12} = D_{g12} \tag{8}$$

such that the closed-loop system is stable and the transfer function T_{zw} from w to z satisfies (2).

For convenience we assume that C_{g2} is of full row rank and has the form

$$C_{g2} = \begin{bmatrix} I_{p_2} & 0 \end{bmatrix} \tag{9}$$

If this is not the case, we can perform a state transformation on x_g such that (9) has the desired form. The output feedback control (4) then becomes

$$\begin{aligned} u &= \begin{bmatrix} C_k & D_k \end{bmatrix} \begin{bmatrix} I_{n_c} & 0 & 0 \\ 0 & I_{p_2} & 0 \end{bmatrix} x \\ &= \begin{bmatrix} C_k & D_k & 0 \end{bmatrix} x = F_d x \end{aligned} \tag{10}$$

where $F_d = \begin{bmatrix} C_k & D_k & 0 \end{bmatrix}$ is a structurally constrained state-feedback gain matrix whose last $n - p_2$ columns are identically zero.

The H_∞ structurally constrained feedback control problem of finding (10) for the generalized system (5) to satisfy (2) is a non-convex programming problem. There is no known closed-form solution to the problem. We approach this problem by formulating a dual design problem, resulting in an optimization problem with quadratic matrix inequality (QMI) constraints. The QMI optimization problem can then be solved iteratively as an LMI problem. Our approach is summarized in Theorems 1 and 2, described below.

We assume that the generalized system (5) satisfies the following assumptions:

(A1) System (5) is stabilizable under the control structure (10).

(A2) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω.

(A3) D_{12} is of full column rank.

Theorem 1: [6] Suppose the generalized plant (5) satisfies Assumptions (A1) - (A3) and D_{12} has the singular value decomposition $D_{12} = U \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V^T$, where U and V are unitary matrices and Σ is a diagonal matrix. Then, for a given $\gamma > 0$, if the algebraic Riccati equation

$$A_H^T X_\infty + X_\infty A_H + X_\infty R_\gamma X_\infty - Q_\gamma = 0 \quad (11)$$

admits a positive semi-definite matrix solution X_∞, where

$$\begin{aligned} A_H &= A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 & (12) \\ R_\gamma &= \gamma^{-2} B_1 B_1^T - B_2 (D_{12}^T D_{12})^{-1} B_2^T & (13) \\ Q_\gamma &= -C_1^T (I - D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T) C_1 & (14) \end{aligned}$$

and the structurally constrained feedback matrix F_d (10) is chosen such that a dual system in the packed matrix form satisfy

$$\left\| \left[\begin{array}{c|c} A_{\text{tmp}} + B_2 F_d & B_1 \\ \hline S_u(F_d - F) & 0 \end{array} \right] \right\|_\infty < \gamma \quad (15)$$

where

$$\begin{aligned} A_{\text{tmp}} &= A + \gamma^{-2} B_1 B_1^T X_\infty & (16) \\ S_u &= \Sigma V^T & (17) \\ F &= -(D_{12}^T D_{12})^{-1} (B_2^T X_\infty + D_{12}^T C_1) & (18) \end{aligned}$$

then the controller $u = F_d x$ (10) is a stabilizing controller satisfying $\| T_{zw} \|_\infty < \gamma$ (2).

In Theorem 1, the H_∞ design problem is to find the control (10) by optimizing (15) instead of $\| T_{zw} \|_\infty$ (2). Thus the optimization of (15) is called a dual-design problem. The result of Theorem 1 arises from the parametrization of all controllers satisfying (2) having the structure F_d. Note that the algebraic Riccati equation (11) arises from an unconstrained H_∞ optimization problem and F (18) is the desired full-state feedback control.

The optimization (15) can be formulated as a QMI problem which is, in general, difficult to solve. However, we introduce a free matrix parameter X to form a second QMI problem which can be solved as a sequence of LMI problems. The required design equations are stated in the following theorem.

Theorem 2: [6] If there exist $M \geq 0$, F_d having the structure (10), and X such that the QMI

$$\begin{bmatrix} \Gamma & \gamma^{-1} M B_1 & \Phi^T S_u^T \\ \gamma^{-1} B_1^T M & -I & 0 \\ S_u \Phi & 0 & -I \end{bmatrix} \leq 0 \quad (19)$$

where $B_s = B_2 S_u^{-1}$, $B_{ss} = B_s B_s^T$, and

$$\Gamma = A_F^T M + M A_F - X^T B_{ss} M - M B_{ss} X + X^T B_{ss} X \quad (20)$$

$$\Phi = F_d - F + S_u^{-1} S_u^{-T} B_2^T M, \quad A_F = A_{\text{tmp}} + B_2 F \quad (21)$$

then the control (10) satisfies $\| T_{zw} \|_\infty < \gamma$ (2).

Theorem 2 is derived based on the bounded real lemma [5]. The QMI (19) points to an iterative approach to solve for F_d, namely, if X is fixed in (20), then (19) reduces to an LMI problem for a given γ in the unknowns F_d and $M \geq 0$. The LMI problem is convex and can be solved, if a feasible solution exists, using existing LMI solvers [7] which use efficient interior point solution techniques. An iterative design procedure is introduced in [6] in which at every iteration, the trace of M is minimized. Then X is set to M and the structure of F_d is updated for the next iteration. When the iterative algorithm converges, the controller parameters C_k and D_k can be obtained from F_d. This iterative method does not have global convergence, but will converge if the initial full-state feedback solution F is close to a local solution. In addition, if the method fails to converge, it does not necessarily mean that there is no solution.

One of the major concerns with state-space design methods is that, in general, it is not possible to directly impose constraints on the controller itself. As a result, the design may provide a controller having poles and zeros in the right half-plane, even though a stable minimum-phase controller exists. In the fixed-pole controller design, we can impose constraints such as positive realness and decentralized structure directly on the controller. For example, if it is desired that the controller be positive real, the LMI constraints [5]

$$\begin{bmatrix} A_k^T P + P A_k & P B_k - C_k^T \\ B_k^T P - C_k & -(D_k + D_k^T) \end{bmatrix} \leq 0 \quad (22)$$

$$P = P^T > 0 \quad (23)$$

can be added to the design formulation, where P is an $n_c \times n_c$ positive definite matrix. Because A_k and B_k are already selected, (22) and (23) are linear in the variables C_k, D_k, and P. Then the design parameters M, $F_d = [\ C_k\ D_k\ 0\ ]$, X, and P need to be selected to satisfy the inequalities (19), (22), and (23). The resulting low-order controller will then be minimum phase having no right-half-plane zeros. This is an important point in power system damping controller design because minimum-phase lead-lag compensators have been shown to provide good performance. If a decentralized controller structure is desired, the controller matrices A_k and B_k can be chosen to have an appropriate block-diagonal form. In the LMI design, the corresponding off-diagonal blocks in C_k and D_k can be set to zero. This decentralized design will be used in the paper.

III. Design Results

This section presents the control design results of the proposed algorithm to the system depicted in Figure 2. The test system is a modified 7-bus equivalent model of the Itaipu generation and AC transmission complex presented in [4]. The reader can refer to [4] for a description of the complete system data.

The system presents a control design challenge, as attempts to stabilize the system using only one power system stabilizer fail. The reason for this limitation is due to the presence of an unstable right-half-plane zero that occurs in the single-input single-output (SISO) transfer function from the AVR reference voltage to the rotor machine speed regardless of the generator considered. The solution to this problem is to consider the closure of a second stabilizer channel. In the coordinated design, we remark that for achieving global controllability, one of the PSSs must be placed at the Itaipu generator.

The modal analysis of the system indicates that there are two interarea modes of interest. Mode 1 behavior has the Southeast (SE) equivalent system oscillating against the Itaipu complex as shown in the mode shape of Figure 3. Mode 2 behavior has the South system (represented by Santiago, Segredo and Areia) oscillating against the Southeast together with Itaipu (Figure 4). The system also presents two local modes of oscillation within the Southern system, Mode 3 represents Areia and Segredo oscillating against Santiago (Figure 5), and finally Mode 4 which represents Areia oscillating against Segredo (Figure 6).

Table I presents five cases, obtained from the same transmission network configuration, where the values of the reactances X_{5-6} and X_{6-7} (connecting Buses 5-6 and Buses 6-7) are varied. The generator and load levels are the same in all cases. Table I also shows the frequencies (Hz) and the damping ratios (%) of Modes 1 and 2 in all five system conditions. Regardless of the system condition, both local modes (Mode 3 and Mode 4) present a damping ratio of about 20% and oscillation frequency of 1.46 Hz. Case #5 shows the weakest operating condition, due to the higher impedance lines.

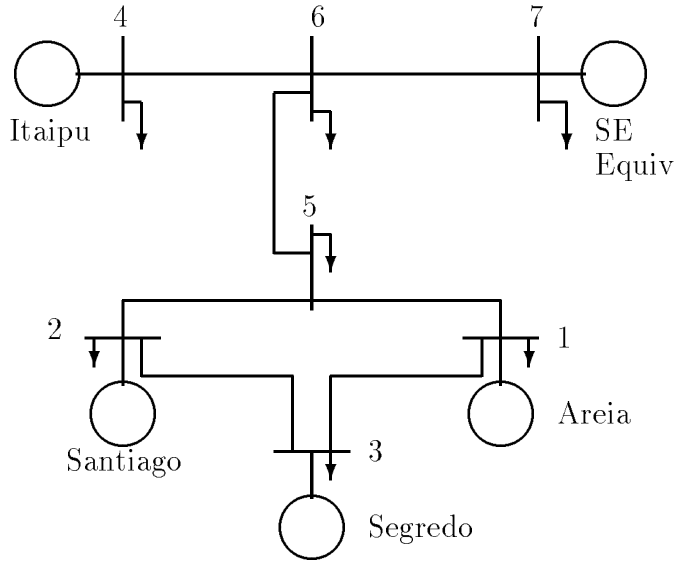

Fig. 2. Study System Configuration

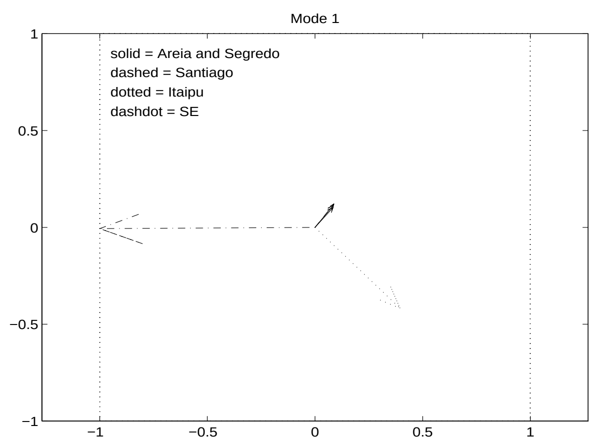

Fig. 3. Rotor-Speed Mode Shape (Mode 1)

Based on the previous system description we assume that there is one PSS at Segredo and another at the Itaipu generator. Both stabilizers are considered for the coordinated design using the technique described in Section II. The reactances of transmission lines 5-6 and 6-7 are considered as the uncertainties in the model, and the design is based on the system Case #5. Figure 7 shows the multivariable pole-zero map (Case #5) when considering PSSs, derived from rotor-speed, at Segredo and Itaipu generators. Note that the zero locations allow damping enhancement for both interarea modes.

Two control designs are compared. Design #1

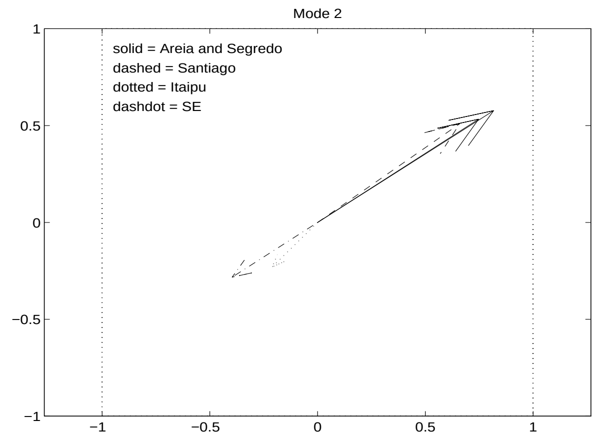

Fig. 4. Rotor-Speed Mode Shape (Mode 2)

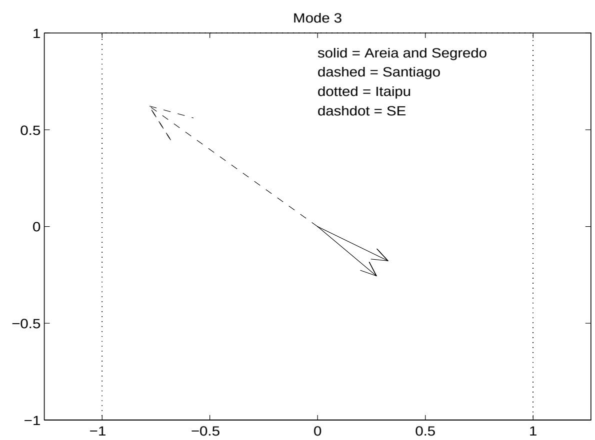

Fig. 5. Rotor-Speed Mode Shape (Mode 3)

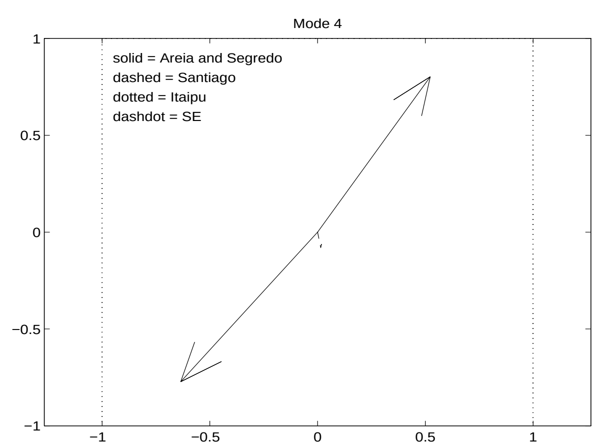

Fig. 6. Rotor-Speed Mode Shape (Mode 4)

TABLE I
Open-loop Operating Conditions

Case	X_{5-6}	X_{6-7}	Mode 1		Mode 2	
#	pu	pu	f(Hz)	ζ(%)	f(Hz)	ζ(%)
1	0.39	0.57	0.86	-11.9	0.94	3.8
2	0.50	0.57	0.86	-12.1	0.92	3.5
3	0.80	0.57	0.85	-12.7	0.88	2.8
4	0.39	0.63	0.84	-13.7	0.93	4.0
5	0.39	0.70	0.80	-16.6	0.93	4.2

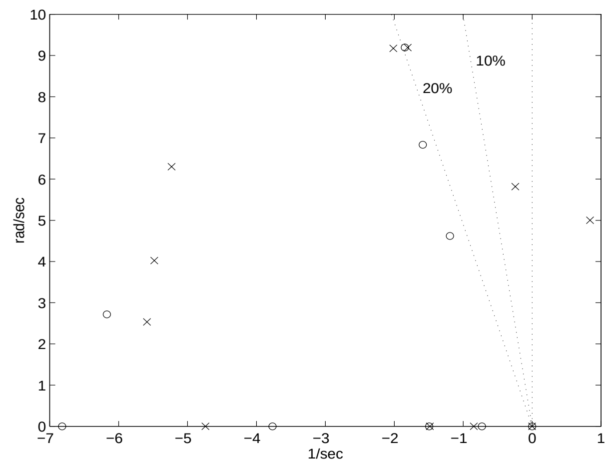

Fig. 7. Multivariable Pole-Zero Map

was taken from [4], and Design #2 was obtained by the LMI algorithm of this paper.

Design #1

$$PSS_3(s) = 10 \times \frac{3s}{1+3s} \times \left(\frac{1+0.3s}{1+0.075s}\right)^2$$

$$PSS_4(s) = 16 \times \frac{3s}{1+3s} \times \left(\frac{1+0.52s}{1+0.065s}\right)^2$$

Design #2

$$PSS_3(s) = 190 \times \frac{3s}{1+3s} \times \frac{s^2+8.0s+36.0}{s^2+27.0s+178}$$

$$PSS_4(s) = 863 \times \frac{3s}{1+3s} \times \frac{s^2+5.6s+5.2s}{s^2+30.8s+237}$$

Table II presents the closed-loop damping ratios (%) for the two least damped modes, which turned out to be always the interarea Mode 1 and either the local Mode 4 or the Itaipu exciter mode. The latter mode is referred as Mode* in Table II. The second and third columns of the table show the damping ratios when the loop is closed with controllers from Design #1. The fourth and fifth columns give the damping ratios when the loop is closed with controllers from Design #2.

TABLE II

Closed-Loop System Damping Ratios (%).

Case #	Design #1 Mode1	Design #1 Mode*	Design #2 Mode1	Design #2 Mode*
1	5.65	10.50	6.86	10.12
2	5.15	10.64	6.49	10.28
3	4.09	10.96	5.71	10.64
4	4.46	10.91	5.83	10.59
5	2.50	11.59	3.94	11.36

To assess the performance of both designs, multivariable root locus for varying gains of the two stabilizers are shown in Figures 8 and 9. Figure 8 is obtained for the base case and Figure 9 is obtained for the weakest case (Case #5). Note the similar performances of both designs, which leads to the conclusion that the LMI-based design for third-order stabilizers has comparable performance to the original design [4] (also of third order).

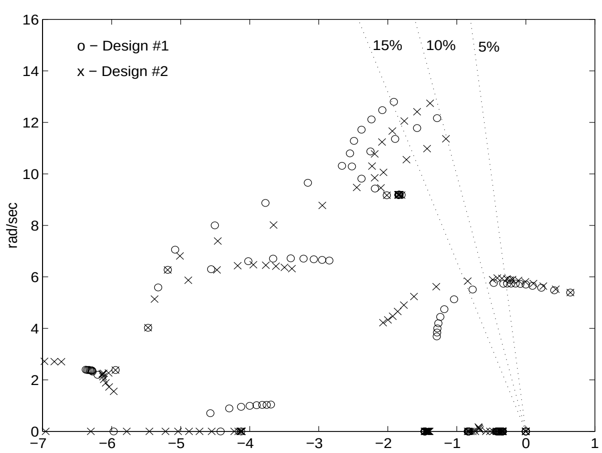

Fig. 8. Root Contour when Varying Gains of the Two Stabilizers at Configuration #1

IV. Conclusions

This paper presented a systematic power system damping control design based on an iterative LMI algorithm, which can accommodate a decentralized control structure once the controller poles are fixed.

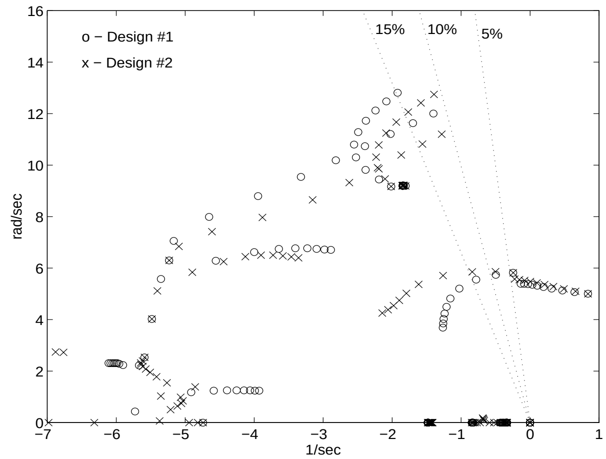

Fig. 9. Root Contour when Varying Gains of the Two Stabilizers at Configuration #5

The performance of the damping controllers designed with the LMI algorithm was comparable to those previously obtained through a more conventional design [4], but of the same order.

It was left open for further investigations the influence of the controller order on system robustness and performance.

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